THE OPTIMIST CLASSES
IIT-JAM TOPPERS

8 AIR

MANOJ KUMAR SINGH

9 AIR

ABHAY

12 AIR

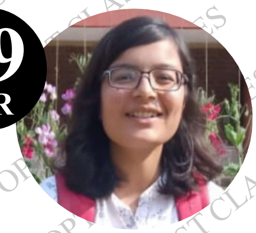
PAWAN

15 AIR

SATYAM

21 AIR

SOUMIL GIRISH SAHU

29 AIR

BHOOMIJA

31 AIR

AKSHIT AGGARWAL

32 AIR

SHIKHAR CHAMOLI

33 AIR

RAVI SINGH ADHIKARI

44 AIR

GAURAV JHA

56 AIR

SWAPNIL JOSHI

65 AIR

LOKESH BHATT

75 AIR

GOPESH VISHVAKARMA

77 AIR

VAIBHAV

80 AIR

SHASWAT CHAMOLI

THE OPTIMIST CLASSES

CSIR-NET-JRF RESULTS 2022

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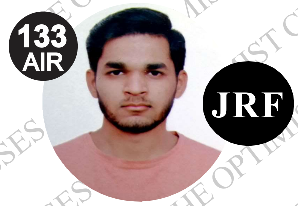


ALANKAR
UP15000162

SAHIL RANA
HR09000108

JAYESTHI
RJ11000161

DASRATH
RJ06000682

VIVEK
UK01000439

UZAIR AHMED
UP02000246

SURYA PRATAP SINGH
RJ06000232

HIMANSHU
UP10000095

CHANDAN
RJ09000159

SAIKHOM JOHNSON
MN01000196

AJAY SAINI
RJ06001744

VIKAS YADAV
RJ06001102

JYOTSNA KOHLI
UK02000262

SHYAM SUNDAR
RJ06000615

THE OPTIMIST CLASSES

THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

CONTACT: 9871044043

GATE PAPER 2005

Q. 1 - .30 : Carry ONE mark each.

Q1. The average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is

(a) 15 (b) 20 (c) 40 (d) 80

Q2. The unit normal to the curve $x^3 y^2 + xy = 17$ at the point (2, 0) is

(a) $\frac{(\hat{i} + \hat{j})}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

Q3. The value of the integral $\int_c \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z| = 4$, is:

(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$

Q4. The determinant of a 3×3 real symmetric matrix is 36. If two of its eigenvalues are 2 and 3 then the third eigenvalue is:

(a) 4 (b) 6 (c) 8 (d) 9

Q5. For a particle moving in a central field

(a) the kinetic energy is a constant of motion (b) the potential energy is velocity dependent

(c) the motion is confined in a plane (d) the total energy is not conserved

Q6. A bead of mass m slides along a straight frictionless rigid wire rotating in a horizontal plane with a constant angular speed ω. The axis of rotation is perpendicular to the wire and passes through one end of the wire. If r is the distance of the mass from the axis of rotation and v is its speed then the magnitude of the Coriolis force is

(a) $\frac{mv^2}{r}$ (b) $\frac{2mv^2}{r}$ (c) $mv\omega$ (d) $2mv\omega$

Q7. If for a system of N particle of different masses $m_1, m_2, ..., m_N$ with position vectors $\vec{r}_1, \vec{r}_2 ...., \vec{r}_N$ and corresponding velocities $\vec{v}_1, \vec{v}_2 ...., \vec{v}_N$, respectively, such that $\sum \vec{v}_i = 0$, then

(a) the total momentum MUST be zero

(b) the total momentum MUST be independent of the choice of the origin

(c) the total force on the system MUST be zero

(d) the total torque on the system MUST be zero

Q8. Although mass-energy equivalence of special relativity allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because

(a) the mass is not conserved (b) the energy is not conserved
(c) the momentum is not conserved (d) the charge is not conserved

Q9. Three infinitely long wires are placed equally apart on the circumference of a circle of radius a, perpendicular to its plane. Two of the wires carry current I each, in the same direction, while the third carries current $2I$ along the direction opposite to the other two. The magnitude of the magnetic induction $\vec{B}$ at a distance r from the centre of the circle, for $r > a$, is

(a) 0 (b) $\frac{2\mu}{\pi}\frac{I}{r}$ (c) $-\frac{2\mu_0}{\pi}\frac{I}{r}$ (d) $\frac{2\mu_0}{\pi}\frac{Ia}{r^2}$

Q10. A solid sphere of radius R carries a uniform volume charge denisty ρ. The magnitude of electric field inside the sphere at a distance r from the centre is:

(a) $\frac{r\rho}{3\varepsilon_0}$ (b) $\frac{R\rho}{3\varepsilon_0}$ (c) $\frac{R^2\rho}{r\varepsilon_0}$ (d) $\frac{R^3\rho}{r^2\varepsilon_0}$

Q11. The electric field $\vec{E}(\vec{r}, t)$ for a circularly polarized electromagnetic wave propagating along the positive z-direction is :

(a) $E_0(\hat{x} + \hat{y}) \exp[i(kz - \omega t)]$ (b) $E_0(\hat{x} + i\hat{y}) \exp[i(kz - \omega t)]$

(c) $E_0(\hat{x} + i\hat{y}) \exp[i(kz + \omega t)]$ (d) $E_0(\hat{x} + \hat{y}) \exp[i(kz + \omega t)]$

Q12. The electric (E) and magnetic (B) field amplitudes associated with an electromagnetic radiation from a point source behave at a distance r from the source as

(a) E = constant, B = constant (b) $E \propto \frac{1}{r}, B \propto \frac{1}{r}$

(c) $E \propto \frac{1}{r^2}, B \propto \frac{1}{r^2}$ (d) $E \propto \frac{1}{r^3}, B \propto \frac{1}{r^3}$

Q13. The parities of the wave functions
(i) cos (kr), and (ii) and tan $h(kx)$ are
(a) (i) odd, (ii) odd (b) (i) even, (ii) even (c) (i) odd, (ii) even (d) (i) even, (ii) odd

Q14. The commutator, $[L_z, Y_{lm}(\theta,\phi)]$ where L_z is the z-component of the orbital angular momentum and $Y_{lm}(\theta,\phi)$ is a spherical harmonic, is:

(a) $l(l+1)\hbar Y_{lm}(\theta,\phi)$ (b) $-m\hbar Y_{lm}(\theta,\phi)$ (c) $m\hbar Y_{lm}(\theta,\phi)$ (d) $+l\hbar Y_{lm}(\theta,\phi)$

Q15. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$, with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigenstates of the system, requires that the constants c_1 and c_2 must satisfy the condition

(a) $|c_1|\cdot|c_2| = 1$ (b) $|c_1| + |c_2| = 1$ (c) $(|c_1| + |c_2|)^2 = 1$ (d) $|c_1|^2 + |c_2|^2 = 1$

Q16. A one-dimensional harmonic oscillator carrying a charge $-q$ is placed in a uniform electric field $\vec{E}$ along positive x-axis. The corresponding Hamiltonian operator is

(a) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$ (b) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

(c) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$ (d) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

Q17. The L_β line of X-rays emitted from an atom with principal quantum numbers $n = 1,2,3,......$, arises from the transition

(a) $n=4 \to n=2$ (b) $n=3 \to n=2$ (c) $n=5 \to n=2$ (d) $n=3 \to n=1$

Q18. For an electron in hydrogen atom, the states are characterized by the usual quantum numbers n, l, m_l. The electric dipole transition between any two states requires that

(a) $\Delta\ell = 0, \Delta m_\ell = 0, \pm 1$ (b) $\Delta\ell = \pm 1, \Delta m_\ell = \pm 1, \pm 2$

(c) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 1$ (d) $\Delta\ell = \pm 1, \Delta m_\ell = 0, \pm 2$

Q19. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is

(a) $pV^{1/3}$ = constant (b) $pV^{2/3}$ = constant

(c) $pV^{4/3}$ = constant (d) $pV^{3/5}$ = constant

Q20. The total number of accessible states of N non intracing particles of spin 1/2 is

(a) 2^N (b) N^2 (c) $2^{N/2}$ (d) N

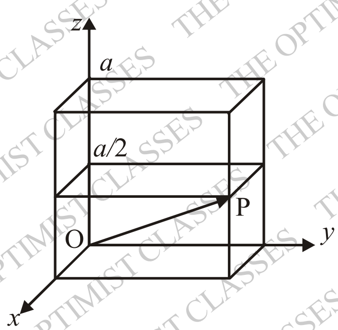
Q21. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is:

(a) $P = \frac{3}{2}\frac{U}{V}$ (b) $P = \frac{2}{3}\frac{U}{V}$ (c) $P = \frac{3}{5}\frac{U}{V}$ (d) $P = \frac{1}{2}\frac{U}{V}$

Q22. A system of non-interacting Fermi particles with Fermi energy E_F has the density of states propor tional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperture $T = 0$ is

(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$ (c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$

Q23. In crystallographic notations the vector $\overline{OP}$ in the cubic cell shown in the figure is



(a) [221] (b) [122] (c) [121] (d) [112]

Q24. Match the following and choose the correct combination

Group- I	**Group-II**
P. Atomic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$	1. Na
Q. Strongly electropositive	2. Si
R. Strongly electronegative	3. Ar
S. Convalent bonding	4. Cl

(a) P-1, Q-2, R-3, S-4 (b) P-3, Q-2, R-4, S-1

(c) P-3, Q-1, R-4, S-2 (d) P-3, Q-4, R-1, S-2

Q25. The evidence for the non-conservation of parity in β-decay has been obtained from the observation that the β intensity

(a) antiparallel to the nuclear spin directions is same as that along the nuclear spin direction

(b) antiparallel to the nuclear spin directions is not the same as that along the nuclear spin direction

(c) shows a continous distributions as a function of momentum
(d) is independent of the nuclear spin direction

Q26. Which of the following expressions for total binding energy B of a nucleus is correct $\left(a_1, a_2, a_3, a_4 > 0\right)$?

(a) $B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

(b) $B = a_1 A + a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

(c) $B = a_1 A + a_2 A^{1/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

(d) $B = a_1 A - a_2 A^{1/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \delta$

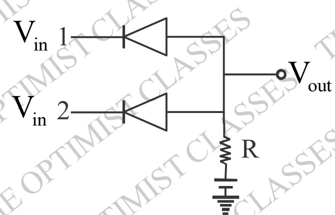
Q27. Which of the following decay is forbidden ?

(a) $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$ (b) $\pi^+ \to \mu^+ + \nu_\mu$
(c) $\pi^+ \to e^+ + \nu_e$ (d) $\mu^- \to e^+ + e^- + e^-$

Q28. With reference to nuclear forces which of the following statements is NOT true ?
The nuclear forces are
(a) short range (b) charge independent
(c) velocity dependent (d) spin independent

Q29. A junction field effect transistor behaves as a
(a) Voltage controlled current source (b) Voltage controlled voltage source
(c) Current controlled voltage source (c) Current controlled current source

Q30. The circuit shown can be used as



(a) NOR gate (b) OR gate (c) NAND gate (d) AND gate

Q.31 - Q.80 : Carry TWO marks each.

Q31. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{F}\right)$ is

(a) 0 (b) $\hat{i}$ (c) $2\hat{j}$ (d) $3\hat{k}$

Q32. All solutions of the equation $e^z = -3$ are
(a) $z = in\pi \ln 3, n = \pm1, \pm2, ...$ (b) $z = \ln 3 + i(2n+1)\pi, n = 0, \pm1, \pm2, ...$
(c) $z = \ln 3 + i2n\pi, n = 0, \pm1, \pm2, ...$ (d) $z = i3n\pi, n = \pm1, \pm2, ...$

Q33. If $\bar{f}(s)$ is the laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constant, is

(a) $\frac{1}{a}\bar{f}(s)$ (b) $\frac{1}{a}\bar{f}(s/a)$ (c) $\bar{f}(s)$ (d) $\bar{f}(s/a)$

Q34. Given the four vectors, $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}$, $u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}$, $u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}$, $u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$, the linearly independent pair is

(a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

Q35. Consider the following function: $f(z) = \frac{\sin z}{z}$. Which of the following statements is are TRUE ?

(a) $z = 0$ is pole of order 1 (b) $z = 0$ is a removable singular point

(c) $z = 0$ is a pole order 3 (d) $z = 0$ is an essential singular point

Q36. Eigenvalues of the matrix

$$\begin{pmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & -2i\\0 & 0 & 2i & 0\end{pmatrix} \text{ are:}$$

(a) – 2, -1, 1, 2 (b) – 1, 1, 0, 2 (c) 1, 0, 2, 3 (d) – 1, 1, 0, 3

Q37. If a particle moves outward in a plane along a curved trajectory described by $r = a\theta,\ \theta = \omega t,$ where a and ω are constant, then its

(a) kinetic energy is conserved (b) angular momentum is conserved

(c) total momentum is conserved (d) radial momentum is conserved

Q38. A circular hoop of mass M and radius a rolls without slipping with constant angular speed ω along the horizontal x -axis in the xy-plane. When the centre of the hoop is at a distance $d = \sqrt{2}a$ from the origin, the magnitude of the total angular momentum of the hoop about the origin is

(a) $Ma^2\omega$ (b) $\sqrt{2}Ma^2\omega$ (c) $2Ma^2\omega$ (d) $3Ma^2\omega$

Q39. Two solid spheres of radius R and mass M each are connected by a thin rigid rod of negligible mass. The distance between the centres is $4R$. The moment of inertia about an axis passing through the centre of symmetry and perpendicular to the line joining the spheres is

(a) $\frac{11}{5}MR^2$ (b) $\frac{22}{5}MR^2$ (c) $\frac{44}{5}MR^2$ (d) $\frac{88}{5}MR^2$

Q40. A car is moving with constant linear acceleration a along horizontal x-axis. A solid sphere of mass M and radius R is found rolling without slipping on the horizontal floor of the car in the same direction as seen from an inerital frame outside the car. The acceleration of the sphere in the inerital frame is

(a) $a/7$ (b) $2a/7$ (c) $3a/7$ (d) $5a/7$

Q41. A rod of length l_0 makes an angle θ_0 with the y-axis in its rest frame, while the rest frame moves to the right along the x-axis with relativistic speed v with respect to the lab frame. If $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$, the angle θ in the lab frame is

(a) $\theta = \tan^{-1}(\gamma \tan\theta_0)$ (b) $\theta = \tan^{-1}(\gamma \cot\theta_0)$

(c) $\theta = \tan^{-1}\left(\frac{1}{\gamma}\tan\theta_0\right)$ (d) $\theta = \tan^{-1}\left(\frac{1}{\gamma}\cot\theta_0\right)$

Q42. A particle of mass m moves in a potential $V(x)=\frac{1}{2}m\omega^2x^2+\frac{1}{2}m\mu v^2$, where x is the position coordinates, v is the speed, and ω and μ are constants. The canonical (conjugate) momentum of the particle is

(a) $p=m(1+\mu)v$ (b) $p=mv$ (c) $p=m\mu v$ (d) $p=m(1-\mu)v$

Q43. Consider the following three independent cases:
(i) Particle A of charge $+q$ moves in free space with a constant velocity $\vec{v}$ ($v <<$ speed of light)
(ii) Particle B of charge $+q$ moves in free space in a circle of radius R with same speed v as in case (i)
(iii) Particle C having charge-q moves as in case (ii)
If the powers radiated by A, B and C are P_A, P_B and P_C respectively, then:

(a) $P_A=0, P_B>P_C$ (b) $P_A=0, P_B=P_C$ (c) $P_A>P_B>P_C$ (d) $P_A=P_B=P_C$

Q44. If the electrostatic potential were given by $\phi=\phi_0\left(x^2+y^2+z^2\right)$, Where ϕ_0 is constant, then the charge density giving rise to the above potential would be:

(a) 0 (b) $-6\phi_0\varepsilon_0$ (c) $-2\phi_0\varepsilon_0$ (d) $-\frac{6\phi_0}{\varepsilon_0}$

Q45. The work done in bringing a charge $+q$ from infinity in free space, to a position at a distance d in front of a semi-infinite grounded metal surface is :

(a) $-\frac{q^2}{4\pi\varepsilon_0(d)}$ (b) $-\frac{q^2}{4\pi\varepsilon_0(2d)}$ (c) $-\frac{q^2}{4\pi\varepsilon_0(4d)}$ (d) $-\frac{q^2}{4\pi\varepsilon_0(6d)}$

Q46. A plane electromagnetic wave travelling in vaccum is incident normally on a non magnetic, non-absorbing medium of refractive index n. The incident (E_i), reflected (E_r) and transmitted (E_t) electric fields are given as, $E_i=E\exp\left[i(kz-\omega t)\right]$, $E_r=E_{0r}\exp\left[i(k_r z-\omega t)\right]$, $E_t=E_{0t}\exp\left[i(k_t z-\omega t)\right]$. If E = 2 V/m and n = 1.5 then the application of appropriate boundary conditions leads to :

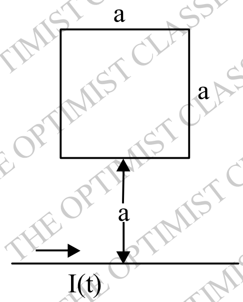
(a) $E_{0r}=-\frac{3}{5}$ V/m, $E_{0t}=\frac{7}{5}$ V/m (b) $E_{0r}=-\frac{1}{5}$ V/m, $E_{0t}=\frac{8}{5}$ V/m

(c) $E_{0r}=-\frac{2}{5}$ V/m, $E_{0t}=\frac{8}{5}$ V/m (d) $E_{0r}=\frac{4}{5}$ V/m, $E_{0t}=\frac{6}{5}$ V/m

Q47. For a vector potential $\vec{A}$, the diveregence of $\vec{A}$ is $\vec{\nabla}.\vec{A}=-\frac{\mu_0}{4\pi}\frac{Q}{r^2}$ where Q is a constant of appropriate dimension. The corresponding scalar potential $\varphi(\vec{r},t)$ that makes $\vec{A}$ and φ Lorentz gauge invariant is :

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{Qt}{r^2}$

Q48. An infinitely long wire carrying a current $I(t)=I_0\cos(wt)$ is placed at a distance a from a square loop of side a as shown in the figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is

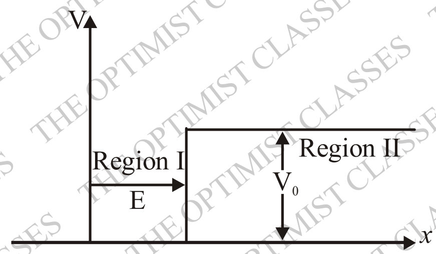


(a) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}\ln 2$ (b) $\frac{\mu_0}{\pi}\frac{aI_0\omega}{R}\ln 2$ (c) $\frac{2\mu_0}{\pi}\frac{aI_0\omega}{R}\ln 2$ (d) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}$

Q49. The de-Broglie wavelength λ for an electron of energy 150 eV is

(a) $10^{-8}\,m$ (b) $10^{-10}\,m$ (c) $10^{-12}\,m$ (d) $10^{-14}\,m$

Q50. A particle is incident with a constant energy E on a one-dimensional potential barrier as shown in the figure. The wave functions in regions I and II are respectively



(a) decaying, oscillatory (b) oscillatory, oscillatory
(c) oscillatory, decaying (d) decaying, decaying.

Q51. The expectation value of the z-coordinates, (z) in the ground state of the hydrogen atom (wavefunction : $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is

(a) a_0 (b) $\frac{a_0}{2}$ (c) $\frac{a_0}{4}$ (d) 0

Q52. The degeneracy of the $n = 2$ level for a three dimensional isotropic oscillator is

(a) 4 (b) 6 (c) 8 (d) 10

Q53. For a spin -1/2 particle, the expectation value of S_x, S_y, S_z. where S_x, S_y and S_z are spin operators, is

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$ (c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

Q54. An atom emits a photon of wavelength $\lambda = 600\,nm$ by transition from an excited state of lifetime 8×10^{-9} s .If $\Delta\nu$ represents the minimum uncertainty in the frequency of the photon, the fractional width $\Delta\nu/\nu$ of the spectral line is of the order of (a)

10^{-4} (b) 10^{-6} (c) 10^{-8} (d) 10^{-10}

Q55. The sodium doublet lines are due to transitions from $^2P_{3/2}$ and $^2P_{1/2}$ levels to $^2S_{1/2}$ level. On application of a weak magnetic field, the total number of allowed transitions becomes

(a) 4 (b) 6 (c) 8 (d) 10

Q56. A three level system of atoms has N_1 atoms in level E_1, N_2 in level E_2, and N_3 in level E_3 $(N_2 > N_1 > N_3$ and $E_1 < E_2 < E_3)$. Laser emission is possible between the levels

(a) $E_3 \to E_1$ (b) $E_2 \to E_1$ (c) $E_3 \to E_2$ (d) $E_2 \to E_3$

Q57. In the Raman scattering experiment, light of frequency v from a laser is scattered by diatomic molecules having moment of inertia I. The typical Raman shifted frequency depends on

(a) v and I (b) only v (c) only I (d) neither v nor I

Q58. For a diatomic moelcule with the vibrational quantum number n and rotational quantum number J, the vibrational level spacing $\Delta E_n = E_n - E_{n-1}$ and the rotational level spacing $\Delta E_J = E_J - E_{J-1}$ are approximately

(a) ΔE_n = constant, ΔE_J = constant (b) ΔE_n = constant, $\Delta E_J \propto J$

(c) $\Delta E_n \propto n, \Delta E_J \propto J$ (d) $\Delta E_n \propto n, \Delta E_J \propto J^2$

Q59. The typical wavelength emitted by diatomic molecules in purely vibrational and purely rotational transition are respectively in the region of

(a) infrared and visible (b) visible and infrared

(c) infrared and microwave (d) microwave and infrared

Q60. In a two electron atomic system having orbital and spin angular momenta ℓ_1, ℓ_2 and $s_1 s_2$ respectively, the coupling strengths are defined as $\Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}, \Gamma_{\ell_1 s_1}, \Gamma_{\ell_2 s_2}, \Gamma_{\ell_1 s_2}$ and $\Gamma_{\ell_2 s_1}$. For the J-J coupling scheme to be applicable, the coupling strenghts MUST satisty the condition.

(a) $\Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2} > \Gamma_{\ell_1 s_1}, \Gamma_{\ell_2 s_2}$ (b) $\Gamma_{\ell_1 s_1}, \Gamma_{\ell_2 s_2} > \Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}$

(c) $\Gamma_{\ell_1 s_2}, \Gamma_{\ell_2 s_1} > \Gamma_{\ell_1\ell_2}, \Gamma_{s_1s_2}$ (d) $\Gamma_{\ell_1 s_2}, \Gamma_{\ell_2 s_1} > \Gamma_{\ell_1 s_1}, \Gamma_{\ell_2 s_2}$

Q61. If the probability that x lies between x and $x + dx$ is $p(x)\,dx = ae^{-ax}\,dx$, where $0 < x < \infty, a > 0$, then the probability that x lies between x_1 and x_2 $(x_2 > x_1)$ is :

(a) $\left(e^{-ax_1} - e^{-ax_2}\right)$ (b) $a\left(e^{-ax_1} - e^{-ax_2}\right)$ (c) $e^{-ax_2}\left(e^{-ax_1} - e^{-ax_2}\right)$ (d) $e^{-ax_1}\left(e^{-ax_1} - e^{-ax_2}\right)$

Q62. If the partition function of a harmonic oscillator wtih frequency ω at a temperature T is $\frac{kT}{\hbar\omega}$, then the free energy of N such independent oscillators is:

(a) $\frac{3}{2}NkT$ (b) $NT\,\text{In}\frac{\hbar\omega}{kT}$ (c) $NkT\,\text{In}\frac{\hbar\omega}{kT}$ (d) $NkT\,\text{In}\frac{\hbar\omega}{2kT}$

Q63. The partition functiion of two Bose particles each of which can occupy any of the two energy levels 0 and ε is:

(a) $1+e^{-2\varepsilon/kT}+2e^{-\varepsilon/kT}$ (b) $1+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (c) $2+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (d) $e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

Q64. A one dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origin is back to origin after N even number of step is:

(a) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$ (c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

Q65. The number of states for a system of N identical free particles in a three dimensional space having total energy between E and E+ $\delta E\,(\delta E \ll E)$, is proportional to

(a) $\left(E^{\frac{3N}{2}-1}\right)\delta E$ (b) $E^{N/2}\delta E$ (c) $NE^{1/2}\delta E$ (d) $E^N\delta E$

Q66. The energy of a ferromagnet as a function of magnetization M is given by

$F(M) = F_0 + 2(T - T_C)M^2 + M^4, F_0 > 0$

The number of minima in the function $F(M)$ for $T > T_C$ is

(a) 0 (b) 1 (c) 3 (d) 4

Q67. For a closed packed BCC structure of hard spheres, the lattice constant a is related to the sphere radius R as

(a) $a = \frac{4R}{\sqrt{3}}$ (b) $a = 4R\sqrt{3}$ (c) $a = 4R\sqrt{2}$ (d) $a = 2R\sqrt{2}$

Q68. An n-type semiconductor has an electron concentration of $3 \times 10^{20}\ \text{m}^{-s}$. If the electron drift velocity is 100 ms^{-1} in an electric field of 200 Vm^{-1}, the conductivity $\left(\text{in } \Omega^{-1}\ \text{m}^{-1}\right)$ of this material is

(a) 24 (b) 36 (c) 48 (d) 96

Q69. Density of states of free electrons in a solid moving with an energy 0.1 eV is given by $2.15 \times 10^{21}\ \text{eV}^{-1}\ \text{cm}^{-3}$. The density of states (in eV^{-1} cm^{-3}) for electrons moving with an energy of 0.4 eV will be

(a) 1.07×10^{21} (b) 1.52×10^{21} (c) 3.04×10^{21} (d) 4.30×10^{21}

Q70. The effective density of states at the condution band edge of Ge is $1.04 \times 10^{19} cm^{-3}$ at room temperature (300K). Ge has an optical bandgap of 0.66eV. The intrinsic carrier concentration (in cm^{-3}) in Ge at room temperature (300K) is approximately

(a) 3×10^{10} (b) 3×10^{13} (c) 3×10^{16} (d) 3×10^{16}

Q71. For a conventional superconductor, which of the following statements is NOT true?

(a) Specific heat is discontinuous at transition temperature T_C

(b) The resistivity falls sharply at T_C

(c) It is diamagnetic below T_C

(d) It is paramagnetic below T_C

Q72. A Nucleus having mass number 240 decays by a emission to the ground state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy (in MeV) of the a particle is :

(a) 5.26 (b) 5.17 (c) 5.13 (d) 5.09

Q73. The threshold temperature above which the thermonuclear reaction

$$^3_2He + ^3_2He \rightarrow ^4_2He + 2^1_1H + 12.86\ \text{MeV}$$

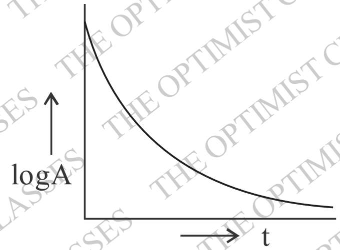
Can occur is $\left(\text{use } e^2/4\pi\varepsilon_0 = 1.44 \times 10^{-15}\ \text{MeV-m}\right)$

(a) 1.28×10^{10} K (b) 1.28×10^{9} K (c) 1.28×10^{8} K (d) 1.28×10^{7} K

Q74. According to shell model, the ground state of $^{15}_{8}O$ nucleus is

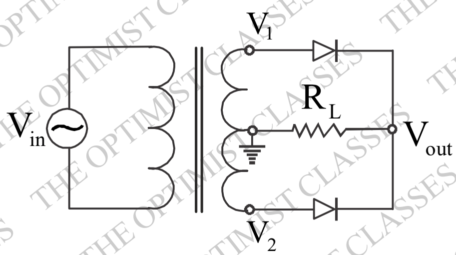
(a) $\frac{3^+}{2}$ (b) $\frac{1^+}{2}$ (c) $\frac{3^-}{2}$ (d) $\frac{1^-}{2}$

Q75. The plot of log$A$ versus time t, where, A is activity, as shown in the figure, corresponds to decay



(a) from only one kind of radioactive nuclei having same half-life.
(b) from only neutron activated nuclei
(c) from a mixture of radioactive nuclei having different half-lives.
(d) which is unphysical.

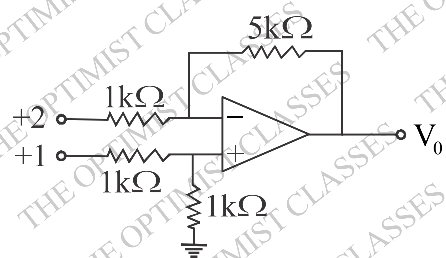
Q76. For the rectifier circuit shown in the figure, the sinusoidal voltage (V_1 or V_2) at the output of the transformer has a maximum value of $10V$. The load resistance R_L is $1\,k\Omega$. If I_{ave} is the averge current through the resistor R_L the circuit corresponds.



(a) Full wave rectifier with $I_{av} = 20/\pi$ mA
(b) Half wave rectifier with $I_{av} = 20/\pi$ mA
(c) Half wave rectifier with $I_{av} = 10/\pi$ mA
(d) Full wave rectifier with $I_{av} = 10/\pi$ mA

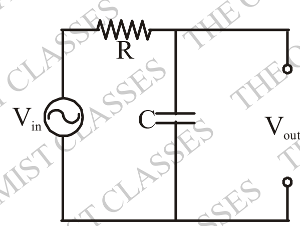
Q77. The Boolean expression : $B(A + B) + A.(\bar{B} + A)$ can be realized using minimum number of
(a) 1 AND gate (b) 2 AND gates (c) 1 OR gate (d) 2 OR gates.

Q78. The output V_0 of the ideal OP-AMP circuit shown in the figure is :



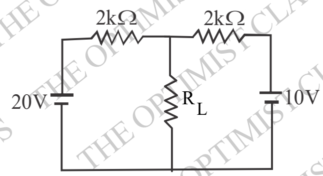
(a) −7V (b) −5V (c) 5 V (d) 7 V

Q79. The circuit shown in the figure can be used as a



(a) high pass filter or differentiator
(b) high pass filter or an integrator
(c) low pass filter or a differentiator
(d) low pass filter or an integrator

Q80. In the circuit shown in the figure the Thevenin voltage V_{Th} and Thevenin resistance R_{Th} as seen by the load resistance $R_L\ (=1k\Omega)$ are respectively.



(a) 15V, 1kΩ (b) 30V, 4kΩ (c) 20V, 2kΩ (d) 10V, 5kΩ

Linkes Answer Questins : Q. 81a to Q. 85.b carry two marks each.
Statement for Linked Answer Q. 27 and Q 28:

For the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$

Q81a. One of the solutions is:

(a) e^x (b) $\ln x$ (c) e^{-x^2} (d) e^{x^2}

Q81b. The second linearly independent solution is:

(a) e^{-x} (b) xe^x (c) x^2e^x (d) x^2e^{-x}

Statement for Linked Answer Q.82a and Q.82b:

The Lagrangian of two coupled oscillators of mass m each is

$$L=\frac{1}{2}m\left(\dot{x}_1^2+\dot{x}_2^2\right)-\frac{1}{2}m\omega_0^2\left(x_1^2+x_2^2\right)+m\omega_0^2\mu x_1x_2$$

Q82a. The equation of motion are

(a) $\ddot{x}_1+\omega_0^2\mu x_1=\omega_0^2\mu x_1, \ddot{x}_2+\omega_0^2x_2=\omega_0^2\mu x_2$ (b) $\ddot{x}_1+\omega_0^2x_1=\omega_0^2\mu x_2,\ \ddot{x}_2+\omega_0^2x_2=\omega_0^2\mu x_1$

(c) $\ddot{x}_1+\omega_0^2\mu x_1=\omega_0^2\mu x_1, \ddot{x}_2+\omega_0^2x_2=-\omega_0^2\mu x_2$ (d) $\ddot{x}_1+\omega_0^2\mu x_1=\omega_0^2\mu x_1,\ \ddot{x}_2+\omega_0^2x_2=\omega_0^2\mu x_1$

Q82b. The normal modes of the system are

(a) $\omega_0\sqrt{\mu^2-1}, \omega_0\sqrt{\mu^2+1}$ (b) $\omega_0\sqrt{1-\mu^2}, \omega_0\sqrt{1+\mu^2}$

(c) $\omega_0\sqrt{\mu-1}, \omega_0\sqrt{\mu+1}$ (d) $\omega_0\sqrt{1-\mu}, \omega_0\sqrt{1+\mu}$

Statement for linked Answer Q. 83a. and Q. 83b :

An infinitely long hollow cylinder of radius R carrying a surface charge density σ is rotated about its cylinderical axis with a constant angular speed ω

Q83a. The magnitude of the surface current is :

(a) $\sigma R\omega$ (b) $2\sigma R\omega$ (c) $\pi\sigma R\omega$ (d) $2\pi\sigma R\omega$

Q83b. The magnitude of vector potential inside the cylinder at a distance from its axis is :

(a) $2\mu_0\sigma R\omega r$ (b) $\mu_0\sigma R\omega r$ (c) $\frac{1}{2}\mu_0\sigma R\omega r$ (d) $\frac{1}{4}\mu_0\sigma R\omega r$

Common data for Q.84a and Q.84b

A particle is scattered by spherically symmetric potential. In the centre of mass (CM) frame the wavefunction of the incoming particle is $\psi = Ae^{ikz}$ where k is the wavevector and A is a constant.

Q84a. If $f(\theta)$ is an angular function then in the asymptotic region the scattered wavefunction has the form

(a) $\frac{Af(\theta)e^{ikr}}{r}$ (b) $\frac{Af(\theta)e^{-ikr}}{r}$ (c) $\frac{Af(\theta)e^{ikr}}{r^2}$ (d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

Q84b. The differential scattering cross section $\sigma(\theta)$ in CM frame is:

(a) $\sigma(\theta) = |A|^2 \frac{|f(\theta)|^2}{r^2}$ (b) $\sigma(\theta) = |A|^2 |f(\theta)|^2$

(c) $\sigma(\theta) = |f(\theta)|^2$ (d) $\sigma(\theta) = |A||f(\theta)|$

Statement for Linked Answer Q. 85(a) and 85(b):

Lead has atomic weight of 207.2 amu and density of 11.35$gm\ cm^{-3}$.

Q85.a Number of atoms per cm^3 for lead is

(a) 1.1×10^{25} (b) 3.3×10^{22} (c) 1.1×10^{22} (d) 3.3×10^{25}

Q85.b If the energy of vacancy formation in lead is 0.55eV/atoms, the number of vacancies/cm^3 at 500K is

(a) 3.2×10^{16} (b) 3.2×10^{19} (c) 9.5×10^{19} (d) 9.5×10^{16}

ANSWER KEY

1.	**(c)**	**2.**	**(d)**	**3.**	**(c)**	**4.**	**(b)**	**5.**	**(c)**
6.	**(d)**	**7.**	**(none)**	**8.**	**(c)**	**9.**	**(a)**	**10.**	**(a)**
11.	**(b)**	**12.**	**(b)**	**13.**	**(d)**	**14.**	**(c)**	**15.**	**(d)**
16.	**(c)**	**17.**	**(a)**	**18.**	**(c)**	**19.**	**(c)**	**20.**	**(a)**
21.	**(b)**	**22.**	**(d)**	**23.**	**(a)**	**24.**	**(c)**	**25.**	**(a)**
26.	**(a)**	**27.**	**(d)**	**28.**	**(d)**	**29.**	**(a)**	**30.**	**(d)**
31.	**(a)**	**32.**	**(b)**	**33.**	**(b)**	**34.**	**(d)**	**35.**	**(b)**
36.	**(a)**	**37.**	**(d)**	**38.**	**(c)**	**39.**	**(c)**	**40.**	**(b)**
41.	**(c)**	**42.**	**(d)**	**43.**	**(b)**	**44.**	**(b)**	**45.**	**(c)**
46.	**(c)**	**47.**	**(d)**	**48.**	**(b)**	**49.**	**(a)**	**50.**	**(c)**
51.	**(d)**	**52.**	**(b)**	**53.**	**(a)**	**54.**	**(c)**	**55.**	**(d)**
56.	**(b)**	**57.**	**(c)**	**58.**	**(b)**	**59.**	**(c)**	**60.**	**(b)**
61.	**(a)**	**62.**	**(c)**	**63.**	**(b)**	**64.**	**(a)**	**65.**	**(c)**
66.	**(b)**	**67.**	**(a)**	**68.**	**(a)**	**69.**	**(d)**	**70.**	**(b)**
71.	**(d)**	**72.**	**(b)**	**73.**	**(a)**	**74.**	**(d)**	**75.**	**(c)**
76.	**(a)**	**77.**	**(c)**	**78.**	**(a)**	**79.**	**(d)**	**80.**	**(a)**
81.a	**(a)**	**81.b**	**(b)**	**82.a**	**(b)**	**82.b**	**(d)**	**83.a**	**(a)**
83.b	**(c)**	**84.a**	**(a)**	**84.b**	**(c)**	**85.a**	**(b)**	**85.b**	**(d)**