THE OPTIMIST CLASSES
IIT-JAM TOPPERS

8 AIR

MANOJ KUMAR SINGH

9 AIR

ABHAY

12 AIR

PAWAN

15 AIR

SATYAM

21 AIR

SOUMIL GIRISH SAHU

29 AIR

BHOOMIJA

31 AIR

AKSHIT AGGARWAL

32 AIR

SHIKHAR CHAMOLI

33 AIR

RAVI SINGH ADHIKARI

44 AIR

GAURAV JHA

56 AIR

SWAPNIL JOSHI

65 AIR

LOKESH BHATT

75 AIR

GOPESH VISHVAKARMA

77 AIR

VAIBHAV

80 AIR

SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022

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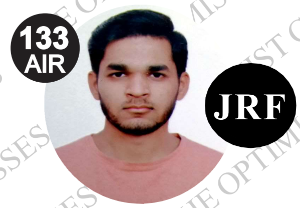

ALANKAR
UP15000162

SAHIL RANA
HR09000108

JAYESTHI
RJ11000161

DASRATH
RJ06000682

VIVEK
UK01000439

UZAIR AHMED
UP02000246

SURYA PRATAP SINGH
RJ06000232

HIMANSHU
UP10000095

CHANDAN
RJ09000159

SAIKHOM JOHNSON
MN01000196

AJAY SAINI
RJ06001744

VIKAS YADAV
RJ06001102

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GATE PAPER 2006

Q.1 - Q.20 :Carry ONE mark each.

Q1. The trace of a 3×3 matrix is 2. Two of its eigenvalues are 1 and 2. The third eigenvalue is

(a) –1 (b) 0 (c) 1 (d) 2

Q2. The value of $\oint_c \vec{A}.d\vec{l}$ along a square loop of side L in a uniform field $\vec{A}$ is

(a) 0 (b) $2LA$ (c) $4LA$ (d) L^2A

Q3. A particle of charge q, mass m and linear momentum $\vec{p}$ enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential ϕ. The Hamiltonian of the particle is

(a) $\frac{p^2}{2m}+q\phi+\frac{A^2}{2m}$ (b) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+q\phi$

(c) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+\vec{p}.\vec{A}$ (d) $\frac{p^2}{2m}q\phi-\vec{p}.\vec{A}$

Q4. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is

(a) circular (b) elliptical (c) parabolic (d) hyperbolic

Q5. In an electromagnetic field, which one of the following remains invariant under Lorentz transformation?

(a) $\vec{E}\times\vec{B}$ (b) $E^2-c^2B^2$ (c) B^2 (d) E^2

Q6. A sphere of radius R has uniform volume charge density. The electric potential at a point $r\left(r<R\right)$ is

(a) due to the charge inside a sphere of radius r only

(b) due to the entire charge of the sphere

(c) due to charge in the spherical shell of inner and outer radii r and R, only

(d) independent of r.

Q7. A free particle is moving in $+x$-direction with a linear momentum p. The wavefunction of the particle normalised in a length L is

(a) $\frac{1}{\sqrt{L}}\sin\frac{p}{\hbar}x$ (b) $\frac{1}{\sqrt{L}}\cos\frac{p}{\hbar}x$ (c) $\frac{1}{\sqrt{L}}e^{-i\frac{p}{\hbar}x}$ (d) $\frac{1}{\sqrt{L}}e^{i\frac{p}{\hbar}x}$

Q8. Which one of the following relations is true for Pauli matrices σ_x,σ_y and σ_z?

(a) $\sigma_x\sigma_y=\sigma_y\sigma_x$ (b) $\sigma_x\sigma_y=\sigma_z$ (c) $\sigma_x\sigma_y=i\sigma_z$ (d) $\sigma_x\sigma_y=-\sigma_y\sigma_x$

Q9. The free energy of a photon gas enclosed ina volume V is given by $F=-\frac{1}{3}a\,\mathrm{VT}^{-4}$, where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is

(a) (b) $\frac{4}{3}aVT^3$ (c) $\frac{1}{3}aT^{-4}$ (d) aVT^{-4}

Q10. The wavefunction of two identical particles in state n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two-particle system is expressed as

(a) $\phi_n(r_1)+\phi_s(r_2)$ (b) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)\phi_s(r_2)+\phi_n(r_2)\phi_s(r_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)\phi_s(r_2)-\phi_n(r_2)\phi_s(r_1)\right]$ (d) $\phi_n(r_1)\phi_s(r_2)$

Q11. The target of an X-ray tube is subjected to an voltage V. The wavelength of the emitted X-ray proportional to

(a) $\frac{1}{\sqrt{V}}$ (b) $\sqrt{V}$ (c) $\frac{1}{V}$ (d) V

Q12. The principal series of spectral lines of lithium is obtained by transitions between
(a) nS and $2P$, $n>2$ (b) nD and $2P$, $n>2$ (c) nP and $2S$, $n>2$ (d) nF and $3D$, $n>3$

Q13. Which one of the following is NOT a correct statement about semiconductors?
(a) The electrons and holes have different mobilities in a semiconductor.
(b) In an n - type semiconductor, the Fermi level lies closer to the conduction band edge.
(c) Silicon is a direct band gap semiconductor.
(d) Silicon has diamond structure.

Q14. Which one of the following axes of rotational symmetry is NOT permissible in single crystals?
(a) two-fold axis (b) three-fold axis (c) four-fold axis (d) five-fold axis

Q15. Weak nuclear forces act on
(a) both hadrons and leptons (b) hadrons only
(c) all particles (d) all charged particle

Q16. Which one of the following disintegration series of the heavy elements will give ^{209}Bi as a stable nucleus?
(a) Thorium series (b) Neptunium series (c) Uranium series (d) Actinium series

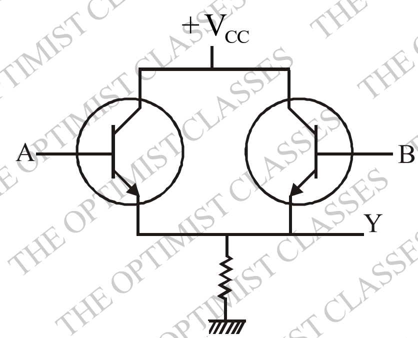
Q17. The order of magnitude of the binding energy per nucleon in a nucleus is
(a) $10^{-5}\ MeV$ (b) $10^{-3}\ MeV$ (c) $0.1\ MeV$ (d) $10\ MeV$

Q18. The interaction potential between two quarks, separated by a distance r inside nucleon, can be described by (a,b and β are positive constants)

(a) $ae^{-\beta r}$ (b) $\frac{a}{r}+br$ (c) $-\frac{a}{r}+br$ (d) $\frac{a}{r}$

Q19. The high input impedance of field effect transistor (FET) amplifier is due to
(a) the pinch-off voltage (b) its very low gate current
(c) the source and drain being far apart (d) the geometry of the EFT

Q20. The circuit shown in the figure function as



(a) an OR gate (b) an AND gate (c) a NOR gate (d) a NAND gate

Q.21 - Q.85 : Carry TWO marks each.

Q21. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\vec{x}$ for a 3-dimensional real space to a 2-dimensional real space. The transformation matrix T is

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Q22. The value of $\oint_S \frac{\vec{r}.d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is

(a) 0 (b) π (c) 4π (d) 8π

Q23. The value of $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is a circle defined by $|z| = 3$, is

(a) $\frac{8\pi i}{3} e^{-2}$ (b) $\frac{8\pi i}{3} e^{-1}$ (c) $\frac{8\pi i}{3} e$ (d) $\frac{8\pi i}{3} e^{2}$

Q24. The kth Fourier component of $f(x) = \delta(x)$ is

(a) 1 (b) 0 (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

Q25. An atom with net magnetic moment $\vec{\mu}$ and net angular momentum $\vec{L}\left(\vec{\mu} = \gamma \vec{L}\right)$ is kept in a uniform magnetic induction $\vec{B} = B_0\hat{k}$. The magnetic moment $\vec{\mu}(= \mu_x)$ is

(a) $\frac{d^2\mu_x}{dt^2} + \gamma B_0 \mu_x = 0$ (b) $\frac{d^2\mu_x}{dt^2} + \gamma B_0 \frac{d\mu_x}{dt} + \mu_x = 0$

(c) $\frac{d^2\mu_x}{dt^2} + \gamma B_0^2 \mu_x = 0$ (d) $\frac{d^2\mu_x}{dt^2} + 2\gamma B_0 \mu_x = 0$

Q26. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a positive constant. In a stationary state, the expectation value of the kinetic energy of the particle is

(a) $\langle T \rangle = \langle V \rangle$ (b) $\langle T \rangle = 2\langle V \rangle$ (c) $\langle T \rangle = 3\langle V \rangle$ (d) $\langle T \rangle = 4\langle V \rangle$

Q27. A particle of mass 2 kg is moving such that at time t, its position, in metre, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at $t = 2s$ about the origin, in $kg\, m^2 s^{-1}$, is

(a) $-40\hat{k}$ (b) $-80\hat{k}$ (c) $80\hat{k}$ (d) $40\hat{k}$

Q28. A system of four particles is in xy-plane of these, two partcles each of mass m are located at (1, 1) and (–1, –1). The remaining two particles each of mass $2m$ are located at (–1, 1) and (1, –1).The xy- component of the moment of inertia tensor of this system of particles is

(a) 10 m (b) –10 m (c) 2 m (d) –2 m

Q29. For the given transformations

(i) $Q = p, P = -q$ (ii) $Q = p,\ P = q$

where p and q are canonically conjugate variables, which one of the following statement is true?

(a) Both (i) and (ii) are canonical (b) Only (i) is canonical
(c) Only (ii) is canonical (d) Neither (i) nor (ii) is canonical

Q30. The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where m_0 is its rest mass. The linear momentum of the particle is

(a) $2m_0c$ (b) $\frac{2m_0}{\sqrt{3}}$ (c) m_0c (d) $\frac{m_0c}{\sqrt{3}}$

Q31. Three point charges q,q and $-2q$ are located at $(0,-a,a)$, $(0,a,a)$ and $(0,0,-a)$, respectively. The net dipole moment of this charge distribution is

(a) $4qa\hat{k}$ (b) $2qa\hat{k}$ (c) $-4qa\hat{i}$ (d) $-2qa\hat{j}$

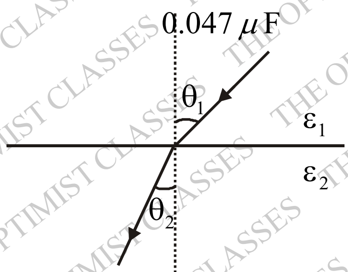
Q32. A long cylindrical kept along z-axis carries a current density $\hat{j} = J_0 r\hat{k}$, where J_0 is a constant and r is the radial distance of from the axis of cylinder. The magnetic induction $\hat{B}$ inside the conductor at a distance d from the axis of the cylinder is

(a) $\mu_0 J_0 \hat{\phi}$ (b) $-\frac{\mu_0 J_0 d}{2}\hat{\phi}$ (c) $\frac{\mu_0 J_0 d^2}{2}\hat{\phi}$ (d) $-\frac{\mu_0 J_0 d^3}{4}\hat{\phi}$

Q33. The vector potential in a region is given as $\vec{A}(x,y,z) = -y\hat{i} + 2x\hat{j}$. The associated magnetic induction is $\vec{B}$ is

(a) $\hat{i}+\hat{k}$ (b) $3\hat{k}$ (c) $-\hat{i}+2\hat{j}$ (d) $-\hat{i}+2\hat{j}$

Q34. At the interface between two linear dilectrics (with dielectric constant ε_1 and ε_2), the electric field lines bend, as shown in the figure. Assume that there are no free charges at the interface. The ratio $\varepsilon_1/\varepsilon_2$ is



(a) $\frac{\tan\theta_1}{\tan\theta_2}$ (b) $\frac{\cos\theta_1}{\cos\theta_2}$ (c) $\frac{\sin\theta_1}{\sin\theta_2}$ (d) $\frac{\cot\theta_1}{\cot\theta_2}$

Q35. Which one of the following sets of Maxwell's equations for time-independent charge density ρ and current density $\hat{j}$ is correct?

(a) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$, $\vec{\nabla}.\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

(b) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$, $\vec{\nabla}.\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\hat{J}$

(c) $\vec{\nabla}.\vec{E} = 0$, $\vec{\nabla}.\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\hat{J}$

(d) $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$, $\vec{\nabla}.\vec{B} = \mu_0\hat{J}$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

Q36. A classical charged particle moving with frequency ω in a circular orbit of radius a, centred at the origin in the xy- plane, electromagnetic radiation. At points $(b,0,0)$ and $(0,0,b)$, where $h >> a$, the electromagnetic waves are

(a) circularly polarized and elliptically polarized, respectively
(b) plane polarized and elliptically polarized,respectively
(c) plane polarized and circularly polarized, respectively
(d) circulary polarized and plane polarized, respectively

Q37. A particle of mass m is represented by the wavefunction $\psi(x) = Ae^{ikx}$, where k is the wavevector and A is a constant. The magnitude of the probability current density of the particle is

(a) $|A|^2 \frac{\hbar k}{m}$ (b) $|A|^2 \frac{\hbar k}{2m}$ (c) $|A|^2 \frac{(\hbar k)^2}{m}$ (d) $|A|^2 \frac{(\hbar k)^2}{2m}$

Q38. A one-dimensional harmonic oscillator is in the state

$$\psi(x) = \frac{1}{\sqrt{14}}\left[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)\right]$$

Where $\psi_0(x), \psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is :

(a) 0 (b) $\frac{3}{\sqrt{14}}$ (c) $\frac{9}{14}$ (d) 1

Q39. The wave function of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t = 0) = \frac{1}{\sqrt{5}}\left[2\psi_0(x) - \psi_1(x)\right]$$

Where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wavefunction of the particle at a time 't' is :

(a) $\frac{1}{\sqrt{5}} e^{i(E_0 E_i)t/2\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (b) $\frac{1}{\sqrt{5}} e^{-iE_0 t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$

(c) $\frac{1}{\sqrt{5}} e^{iE_0 t/\hbar}\left[2\psi_0(x) - \psi_1(x)\right]$ (d) $\frac{1}{\sqrt{5}}\left[2\psi_0(x)^{-iE_0 t/\hbar} - \psi_1(x)^{-iE_1 t/\hbar}\right]$

Q40. The commutator $[L_x, y]$, where L_x is the x-component of the angular momentum operator and y is the y-component of the position operator, is equal to

(a) 0 (b) $i\hbar x$ (c) $i\hbar y$ (d) $i\hbar z$

Q41. In hydro genic states, the probability of finding an electron at $r = 0$ is

(a) zero in state $\phi_{1s}(r)$ (b) non-zero in state $\phi_{1s}(r)$

(c) zero in state $\phi_{2s}(r)$ (d) zero in state $\phi_{2p}(r)$

Q42. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperatures of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contact. At equilibrium, the change in entropy is

(a) $\frac{3}{2}Nk_B \ln\left[\frac{T_1^2+T_2^2}{4T_1T_2}\right]$ (b) $Nk_B \ln\left[\frac{T_2}{T_1}\right]$

(c) $\frac{3}{2}Nk_B \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (d) $2Nk_B$

Q43. The internal energy of n moles of a gas is given $E=\frac{3}{2}nRT-\frac{a}{V}$, where V is the volume of the gas at temperature T and a is a positive constant. One mole of the gas in state (T_1, V_1) is allowed to expand adiabatically into vacuum to a final state (T_2, V_2). The temperature T_2 is

(a) $T_1+Ra\left(\frac{1}{V_2}+\frac{1}{V_1}\right)$ (b) $T_1-\frac{2}{3}Ra\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

(c) $T_1+\frac{2}{3}Ra\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$ (d) $T_1-\frac{1}{3}Ra\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

Q44. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is

(a) $\frac{1}{2}k_BT$ (b) k_BT (c) $\frac{3}{2}k_BT$ (d) $3k_BT$

Q45. A monatomic crystalline solid comprises of N atoms, out of which n atoms are in interstitial positions. If available interstitial sites are N', then number of possible microstates is

(a) $\frac{(N'+n)!}{n!N!}$ (b) $\frac{N!}{n!(N'+n)!}\frac{N!}{n!(N'+n)!}$

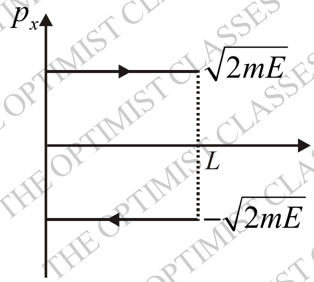
(c) $\frac{N!}{n!(N'-n)!}$ (d) $\frac{N!}{n!(N'-n)!}\frac{N!}{n!(N'-n)!}$

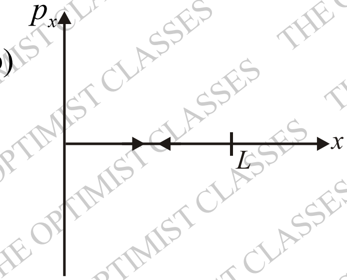
Q46. A system of n localized, non-interacting spin $-\frac{1}{2}$ ions of magnetic moment μ each is kept in external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the sytem is

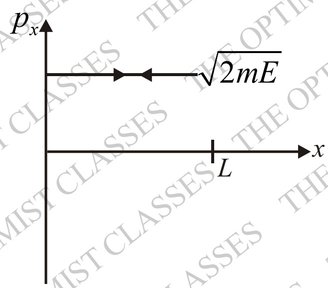
(a) $Nk_BT\ln\left(\cosh\frac{\mu H}{k_BT}\right)$ (b) $-Nk_BT\ln\left(2\cosh\frac{\mu H}{k_BT}\right)$

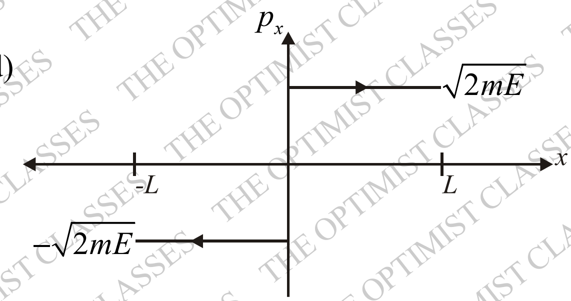
(c) $Nk_BT\ln\left(2\cosh\frac{\mu H}{k_BT}\right)$ (d) $-Nk_BT\ln\left(2\sinh\frac{\mu H}{k_BT}\right)$

Q47. The phase diagram of a free particle of mass m kinetic energy E, moving in one-dimensional box with perfectly elastic walls at $x = 0$ and $x = L$, is given by

(a)


(b)


(c)


(d)


Q48. In the microwave spectrum of identical rigid diatomic molecules, the separartion between the spectral lines is recorded to be $0.7143\,cm^{-1}$. The moment of inertia of the molecules, in $kg\,m^2$, is

(a) 2.3×10^{-36} (b) 2.3×10^{-40} (c) 7.8×10^{-42} (d) 7.8×10^{-46}

Q49. Which one of the following electronics transitions in Neon is NOT responsible for LASER action in a helium - neon laser?

(a) $6s \rightarrow 5p$ (b) $5s \rightarrow 4p$ (c) $5s \rightarrow 3p$ (d) $4s \rightarrow 3p$

Q50. In the linear Stark effect, the application of an electric field

(a) completely lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into four levels

(b) partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into three levels

(c) partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into two levels

(d) does not affect the $n = 2$ levels

Q51. In hyperfine interaction, there is coupling between the electron angular momentum $\vec{J}$ and nuclear angular momentum $\vec{I}$, forming resultant angular momentum $\vec{F}$. The selection rules for the corresponding quantum number F in hyperfine transitions are

(a) $\Delta F = \pm 2$ only (b) $\Delta F = \pm 1$ only (c) $\Delta F = 0, \pm 1$ (d) $\Delta F = \pm 1, \pm 2$

Q.52 A vibrational electronic spectrum of homonuclear binary molecules, involving electronic graound state ε'' and excited ε', exhibits a continuum at $\bar{\nu}\,cm^{-1}$. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by $E_{ex}\,cm^{-1}$, then dissociation energy of the molecule in the ground state is

(a) $\frac{(\bar{v}+E_{ex})}{2}$ (b) $\frac{(\bar{v}-E_{ex})}{2}$ (c) $(\bar{v}-E_{ex})$ (d) $\sqrt{(\bar{v}^2-E_{ex}^2)}$

Q53. The NMR spectrum of ethanol (CH_3CH_2OH) comprises of three bunches of spectral lines. The number of spectral lines in the bunch corresponding to CH_2 group is

(a) 1 (b) 2 (c) 3 (d) 4

Q54. The energy $E(\vec{k})$ of electrons of wavefactor $\vec{k}$ in a solid is given by $E(\vec{k}) = Ak^2 + Bk^4$, where A and B are constants. The effective mass of the electron at $|\vec{k}| = k_0$ is

(a) Ak_0^2 (b) $\frac{\hbar^2}{2A}$ (c) $\frac{\hbar^2}{2A+12Bk_0^2}$ (d) $\frac{\hbar^2}{Bk_0^2}$

Q55. Which one of the following statements is NOT correct about the Brillouin zones (*BZ*) of a square lattices with constants *a*?

(a) The first *BZ* is a square of side $2\pi/a$ in $k_x - k_y$ plane

(b) The areas of the first *BZ* and third *BZ* are the same

(c) The *k*-points are equidistant in k_x as well as in k_y directions

(d) The area of the second *BZ* is twice that of the first *BZ*

Q56. In a crystal of *N* primitive cells, each cell contains two monovalent atoms. The highest occupied energy band of the crystal is

(a) one-fourth filled (b) one-third filled (c) half filled (d) completely filled

Q57. If the number density of a free electron gas changes from 10^{28} to 10^{26} electrons / m^3, the value of plasma frequency (in *Hz*) changes from 5.7×10^{15} to

(a) 5.7×10^{13} (b) 5.7×10^{14} (c) 5.7×10^{16} (d) 5.7×10^{17}

Q58. Which one of the following statements about superconductors is NOT true?

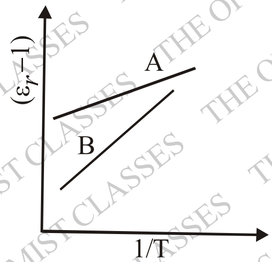
(a) A type I superconductor is completely diamagnetic

(b) A type II superconductor exhibits Meissner effect upto the second critical magnetic field (H_{c_2}).

(c) A type II superconductor exhibits zero resistance upto the second critical magnetic field

(d) Both type I and II superconductors exhibits sharp fall in resistance at the superconducting transition tem perature

Q59. Two dielectric materials *A* and *B* exihibit both ionic and orientational polarizabilities. The variation of their susceptibities $\chi(=\varepsilon_r,-1)$ with temperature *T* is shown in the figure, where ε_r is the relative dielectric constant. It can be inferred from the figure that



(a) A is more polar and it has a smaller value of ionic polarizability thab that of B

(b) A is more polar and it has a higher value of ionic polarizability than that of B

(c) B is more polar and it has a higher value of ionic polarizability than that of A

(d) B is more polar and it has a smaller value of ionic polarizability than that of A

Q60. The experimentally measured spin *g* factors of proton and a neutron indicate that

(a) Both proton and neutron are elementary point particles

(b) Both neutron and proton are elementary point particles

(c) While proton is an elementary point particle, neutron is not

(d) While neutron is an elementary point particle, proton is not

Q61. By capuring an electron, ${}^{54}_{25}Mn_{29}$ transition from ${}^{54}_{25}Cr_{30}$ releasing

(a) a neutrino (b) an antineutrino (c) an α -particle (d) a positron

Q62. Which one of the following nuclear processes is forbidden?

(a) $\overline{v} + p \to n + e^+$

(b) $\pi^- \to e^- + v_e + \pi^0$

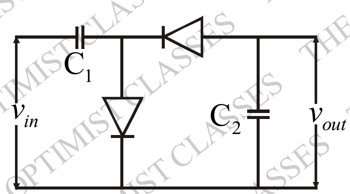
(c) $\pi^- + p^- \to n + K^+ + K^-$

(d) $\mu^- \to e^- + \overline{v}_e + v_\mu$

Q63. To explain the observed magnetic moment of deutron $(0.8574\mu_N)$, its ground state wavefunction is taken to be an admixture of *S* and *D* states. The expectation values of the *z*-component of the magnetic moment in pure *S* and pure *D* states are $0.8797\mu_N$ and $0.3101\mu_N$ respectively. The contribution of the *D* state to the mixed ground state is approximately

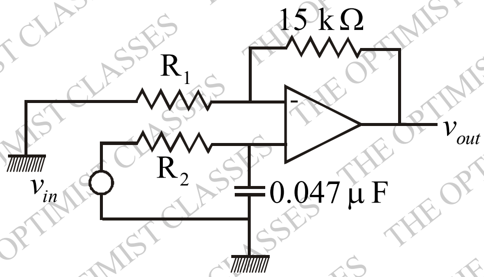
(a) 40% (b) 4% (c) 0.4% (d) 0.04%

Q64. A sinusoidal input voltage v_{in} of frequency ω is fed to the circuit shown in the figure, where $C_1 >> C_2$. If v_m is the peak value of the input voltage, then output voltage (v_{out}) is



(a) $2v_m$ (b) $2v_0 \sin \omega t$ (c) $\sqrt{2}v_m$ (d) $\frac{v_m}{2} \sin \omega t$

Q65. The low-pass active filter shown in the figure has a cut-off frequency of 2 kHz and a pass band gain of 1.5. The values of the resistors are



(a) $R_1 = 10\,k\Omega;\ R_2 = 1.3\Omega$ (b) $R_1 = 30\,k\Omega;\ R_2 = 1.3\Omega$

(c) $R_1 = 10\,k\Omega;\ R_2 = 1.7\,\Omega$ (d) $R_1 = 30\,k\Omega;\ R_2 = 1.7\,\Omega$

Q66. In order to obtain a solution of the differential equation

$$\frac{d^2 v}{dt^2} - 2\frac{dv}{dt} + v_1 = 0,$$

involving voltages $v(t)$ and v_1, an operational amplifier (Op-Amp) circuit would require at least

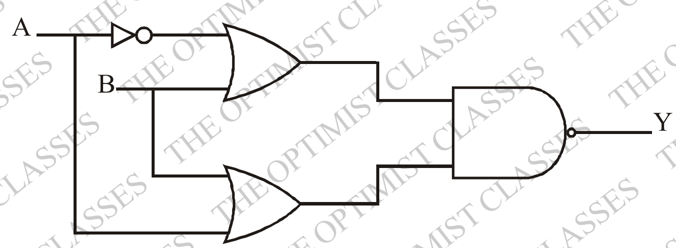
(a) two Op-Amp integrators and one Op-Amp adder

(b) two Op-Amp differentiators and one Op-Amp adder

(c) one Op-Amp ingrators and one Op-Amp adder

(d) one Op-Amp integrator, one Op-Amp differentiator and one Op-Amp adder

Q67. In the given digital logic circuit, A and B form the input. The output Y is

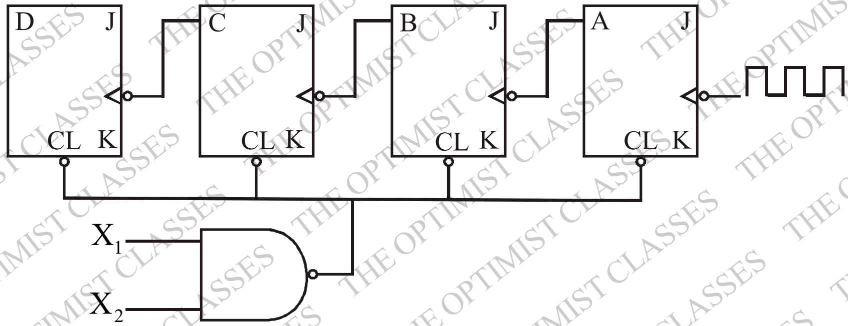


(a) $Y = \overline{A}$ (b) $Y = A\overline{B}$ (c) $Y = A \oplus B$ (d) $Y = \overline{B}$

Q68. The largest analog output voltage from a 6-bit digital to analog converter (DAC) which produces 1.0V output for a digital input of 010100, is

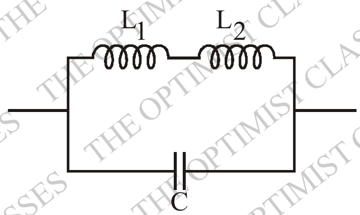
(a) 1.6 V (b) 2.9 V (c) 3.15 V (d) 5.0 V

Q69. A ripple counter designed with JK flip-flops provided with CLEAR (CL) input is shown in the figure. In order that this circuit functions as a MOD-12 counter, the NAND gate input (X_1 and X_2) should be



(a) A and C (b) A and D (c) B and D (d) C and D

Q70. The tank circuit of a Hartley oscillator is shown in the figure. If M is the mutual induactance between the inductors, the oscillation frequency is



(a) $\frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}}$ (b) $\frac{1}{2\pi\sqrt{(L_1+L_2-2M)C}}$

(c) $\frac{1}{2\pi\sqrt{(L_1+L_2+M)C}}$ (d) $\frac{1}{2\pi\sqrt{(L_1+L_2-M)C}}$

Common Data for Q.71, Q.72 and Q.73 :

An unperturbed two level system has energy eigenvalue E_1 and E_2, and eigen function $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.

When perturbed, its Hamiltonian is represented by $\begin{pmatrix}E_1 & A\\A^* & E_2\end{pmatrix}$

Q71. The first-order correct to E_1 is

(a) $4A$ (b) $2A$ (c) A (d) 0

Q72. The second-order correction to E_1 is

(a) 0 (b) A (c) $\frac{A^2}{E_2-E_1}$ (d) $\frac{A}{E_1-E_2}$

Q73. The first-order correction to the eigenfunction $\begin{pmatrix}1\\0\end{pmatrix}$ is;

(a) $\begin{pmatrix}0\\A^*/(E_1-E_2)\end{pmatrix}$ (b) $\begin{pmatrix}0\\1\end{pmatrix}$ (c) $\begin{pmatrix}A^*/(E_1-E_2)\\0\end{pmatrix}$ (d) $\begin{pmatrix}1\\1\end{pmatrix}$

Common data for Q.74 and Q.75.

One of the eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5.

Q74. The other two eigenvalues are

(a) 0 and 0 (b) 1 and 1 (c) 1 and –1 (d) –1 and –1

Q75. The normalized eigenvector corresponding to the eigenvalue 5 is

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Linked Answer Questions : Q. 76 to Q.85 carry two marks each.

Statement for Linked Answer Q.76 to Q.77 :

The powder diffraction pattern of a body centred cubic crystal is recorded by using $Cu\ K_\alpha$ X-rays of wave-length 1.54Å.

Q76. If the (002) planes diffract at $60°$, then lattice parameter is

(a) 2.67Å (b) 3.08 Å (c) 3.56 Å (d) 5.34 Å

Q77. Assuming the atomic mass of the consituent atoms to be 50.94*amu*, then density of the crystal in units of $kg\ m^{-3}$ is

(a) 3.75×10^3 (b) 4.45×10^3 (c) 5.79×10^3 (d) 8.89×10^3

Statements for Linked Answer Q.78 and Q.79 :

A particle of mass m is constrained to move in a vertical plane along a trajectory given by $x = \text{A}\cos\theta$, $y = \text{A}\sin\theta$ where A is a constant.

Q78. The Lagrangian of the particle is

(a) $\frac{1}{2}mA^2\dot{\theta}^2 - mgA\cos\theta$ (b) $\frac{1}{2}mA^2\dot{\theta}^2 - mgA\sin\theta$

(c) $\frac{1}{2}mA^2\dot{\theta}^2$ (d) $\frac{1}{2}mA^2\dot{\theta}^2 + mgA\cos\theta$

Q79. The equation of motion of the particle is

(a) $\ddot{\theta} - \frac{g}{A}\cos\theta = 0$ (b) $\ddot{\theta} + \frac{g}{A}\sin\theta = 0$

(c) $\ddot{\theta} = 0$ (d) $\ddot{\theta} - \frac{g}{A}\sin\theta = 0$

Statements for Linked Answer Q.80 and Q.81 :

A dielectric sphere of radius R carries polarization $\vec{P} = kr^2\hat{r}$, where r is the distance from the centre and k is a constant. In the spherical polar coordinates system, $\hat{r}, \hat{\theta}$ and $\hat{\varphi}$ are the unit vectors.

Q80. The bound volume charge density inside the sphere at a distance r from the centre is

(a) $-4kR$ (b) $-4kr$ (c) $-4kr^2$ (d) $-4kr^3$

Q81. The electric field inside the sphere at a distance d from the centre is

(a) $\frac{-kd^2}{\varepsilon_0}\hat{r}$ (b) $\frac{-k\text{R}^2}{\varepsilon_0}\hat{r}$ (c) $\frac{-kd^2}{\varepsilon_0}\hat{\theta}$ (d) $\frac{-k\text{R}^2}{\varepsilon_0}\hat{\theta}$

Statements for Linked Answer Q.82 and Q.83 :

Consider Fermi theory of β-decay.

Q82. The number of final states of electrons corresponding to momenta between p and $p + dp$ is

(a) independent of p
(b) proportional to pdp
(c) proportional to $p^2 dp$
(d) proportional to $p^3 dp$

Q83. The number of emitted electrons with momentum p and energy E, in the allowed approximation, is proportional to (E_0 is the total energy given up by the nucleus).

(a) $(E_0 - E)$
(b) $p(E_0 - E)$
(c) $p^2(E_0 - E)^2$
(d) $p(E_0 - E)^2$

Statement for Linked Answer Question 84 and 85 :

Consider a radiation cavity of volume V at temperature T.

Q84. The density of states at energy E of the quantized radiation (photons) is

(a) $\frac{8\pi V}{h^3 c^3} E^2$
(b) $\frac{8\pi V}{h^3 c^3} E^{3/2}$
(c) $\frac{8\pi V}{h^3 c^3} E$
(d) $\frac{8\pi V}{h^3 c^3} E^{1/2}$

Q85. The average number of photons in equilibrium inside the cavity is proportional to

(a) T
(b) T^2
(c) T^3
(d) T^4

ANSWER KEY

1.	**(a)**	**2.**	**(a)**	**3.**	**(b)**	**4.**	**(d)**	**5.**	**(b)**
6.	**(b)**	**7.**	**(d)**	**8.**	**(c)**	**9.**	**(a)**	**10.**	**(d)**
11.	**(c)**	**12.**	**(c)**	**13.**	**(c)**	**14.**	**(d)**	**15.**	**(c)**
16.	**(b)**	**17.**	**(d)**	**18.**	**(c)**	**19.**	**(b)**	**20.**	**(a)**
21.	**(a)**	**22.**	**(c)**	**23.**	**(a)**	**24.**	**(c)**	**25.**	**(c)**
26.	**(a)**	**27.**	**(b)**	**28.**	**(c)**	**29.**	**(b)**	**30.**	**(d)**
31.	**(a)**	**32.**	**(c)**	**33.**	**(b)**	**34.**	**(a)**	**35.**	**(b)**
36.	**(c)**	**37.**	**(a)**	**38.**	**(c)**	**39.**	**(d)**	**40.**	**(d)**
41.	**(a, c, d)**	**42.**	**(c)**	**43.**	**(c)**	**44.**	**(b)**	**45.**	**(d)**
46.	**(b)**	**47.**	**(a)**	**48.**	**(d)**	**49.**	**(a)**	**50.**	**(b)**
51.	**(c)**	**52.**	**(c)**	**53.**	**(d)**	**54.**	**(c)**	**55.**	**(d)**
56.	**(c)**	**57.**	**(b)**	**58.**	**(b)**	**59.**	**(d)**	**60.**	**(b)**
61.	**(a)**	**62.**	**(b)**	**63.**	**(b)**	**64.**	**(a)**	**65.**	**(d)**
66.	**(a)**	**67.**	**(d)**	**68.**	**(c)**	**69.**	**(d)**	**70.**	**(a)**
71.	**(d)**	**72.**	**(d)**	**73.**	**(a)**	**74.**	**(c)**	**75.**	**(d)**
76.	**(c)**	**77.**	**(a)**	**78.**	**(a)**	**79.**	**(d)**	**80.**	**(b)**
81.	**(a)**	**82.**	**(c)**	**83.**	**(b)**	**84.**	**(a)**	**85.**	**(c)**