

THE OPTIMIST CLASSES

IIT-JAM TOPPERS



8
AIR
MANOJ KUMAR SINGH



9
AIR
ABHAY



12
AIR
PAWAN



15
AIR
SATYAM



21
AIR
SOUMIL GIRISH SAHU



29
AIR
BHOOMIJA



31
AIR
AKSHIT AGGARWAL



32
AIR
SHIKHAR CHAMOLI



33
AIR
RAVI SINGH ADHIKARI



44
AIR
GAURAV JHA



56
AIR
SWAPNIL JOSHI



65
AIR
LOKESH BHATT



75
AIR
GOPESH VISHVAKARMA



77
AIR
VAIBHAV



80
AIR
SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022



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UP15000162



SAHIL RANA
HR09000108



JAYESTHI
RJ11000161



DASRATH
RJ06000682



VIVEK
UK01000439



UZAIR AHMED
UP02000246



SURYA PRATAP SINGH
RJ06000232



HIMANSHU
UP10000095



CHANDAN
RJ09000159



SAIKHOM JOHNSON
MN01000196



AJAY SAINI
RJ06001744



VIKAS YADAV
RJ06001102



JYOTSNA KOHLI
UK02000262



SHYAM SUNDAR
RJ06000615

THE OPTIMIST CLASSES

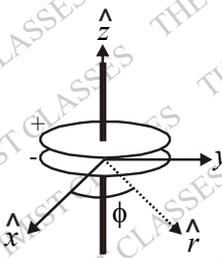
AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

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GATE PAPER 2008

Q.1-Q.20 : Carry ONE mark each.

- Q1. For arbitrary matrices E, F, G and H , if $EF - FE = 0$ then $\text{Trace}(EFGH)$ is equal to
 (a) $\text{Trace}(HGFE)$ (b) $\text{Trace}(E) \cdot \text{Trace}(F) \cdot \text{Trace}(G) \cdot \text{Trace}(H)$
 (c) $\text{Trace}(GFEH)$ (d) $\text{Trace}(EGHF)$
- Q2. An unitary matrix $\begin{pmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{pmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is
 (a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$ (b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$ (c) $\begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}$ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$
- Q3. The curl of a vector field \vec{F} is $2\hat{x}$. Identify the appropriate vector field \vec{F} from the choices given below
 (a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$
 (c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2\hat{x} + 5y\hat{z}$
- Q4. A rigid body is rotating about its centre of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is \vec{L} , the rate of change of its kinetic energy is
 (a) $\frac{1}{2} \vec{\tau} \cdot \vec{\omega}$ (b) $\frac{1}{2} \vec{L} \cdot \vec{\omega}$ (c) $\frac{1}{2} (\vec{\tau} \cdot \vec{\omega} + \vec{L} \cdot \vec{\alpha})$ (d) $\frac{1}{2} \vec{L} \cdot \vec{\alpha}$
- Q5. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ . The number of generalized coordinates required to describe the motion of this system is
 (a) 1 (b) 2 (c) 4 (d) 6
- Q6. A parallel plate capacitor is being discharged. What is the direction of the energy flow in terms of the Poynting vector in the space between the plates?



- (a) Along the wire in positive z axis (b) Radially inward ($-\hat{r}$)
 (c) Radially outward (\hat{r}) (d) Circumferential (ϕ)

Q7. Unpolarized light falls from an air to a planar air-glass interface (refractive index of glass is 1.5) and the reflected light is observed to be plane polarized. The polarization vector and the angle of incidence θ_i are

- (a) perpendicular to the plane of incidence and $\theta_i = 42^\circ$
 (b) parallel to the plane of incidence and $\theta_i = 56^\circ$
 (c) perpendicular to the plane of incidence and $\theta_i = 56^\circ$
 (d) parallel to the plane of incidence and $\theta_i = 42^\circ$

Q8. A finite wave train, of an unspecified nature, propagates along the positive x -axis with a constant speed v and without any change of shape. The differential equation among the four listed below, whose solution it must be, is

- (a) $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2}\right) \Psi(x, t) = 0$ (b) $\left(\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2}\right) \Psi(\vec{r}, t) = 0$
 (c) $\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - i\hbar \frac{\partial}{\partial t}\right) \Psi(x, t) = 0$ (d) $\left(\nabla^2 + a \frac{\partial}{\partial t}\right) \Psi(\vec{r}, t) = 0$

Q9. Let $|\Psi_0\rangle$ denote the ground state of hydrogen atom. Choose the correct statement from those given below:

- (a) $[L_x, L_y]|\Psi_0\rangle = 0$ (b) $J^2|\Psi_0\rangle = 0$
 (c) $\vec{L} \cdot \vec{S}|\Psi_0\rangle \neq 0$ (d) $[S_x, S_y]|\Psi_0\rangle = 0$

Q10. Thermodynamic variables of system can be volume V , pressure P , temperature T , number of particles N , internal energy E and chemical potential μ , etc. For a system to be specified by microcanonical (MC), Canonical (CE) and Grand Canonical (GC) ensembles, the parameters required for the respective ensembles are

- (a) $MC : (N, V, T); CE : (E, V, N); GC : (V, T, \mu)$
 (b) $MC : (E, V, N); CE : (N, V, T); GC : (V, T, \mu)$
 (c) $MC : (V, T, \mu); CE : (N, V, T); GC : (E, V, N)$
 (d) $MC : (E, V, N); CE : (V, T, \mu); GC : (N, V, T)$

Q11. The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is:

- (a) the entropy of solid is zero in this temperature region.
 (b) the entropy increases when the system goes from liquid to solid phase in this temperature region.
 (c) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature.
 (d) the change in entropy is zero in the liquid-to-solid transition region.

Q12. The radial wave function of the electrons in the state of $n = 1$ and $l = 0$ in a hydrogen atom is

$$R_{10} = \frac{2}{a_0^{3/2}} \exp\left(-\frac{r}{a_0}\right)$$

where a_0 is the Bohr radius. The most probable value of r for an electron is

- (a) a_0 (b) $2a_0$ (c) $4a_0$ (d) $8a_0$

Q13. The last two terms of the electronic configuration of manganese (Mn) atom is $3d^5 4s^2$. The term factor of Mn^{4+} ion is

- (a) ${}^4D_{1/2}$ (b) ${}^4F_{3/2}$ (c) ${}^3F_{9/2}$ (d) ${}^3D_{7/2}$

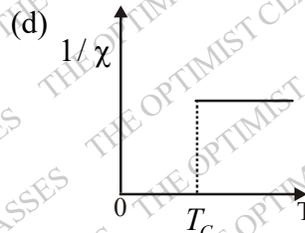
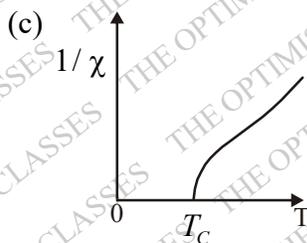
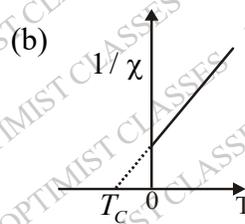
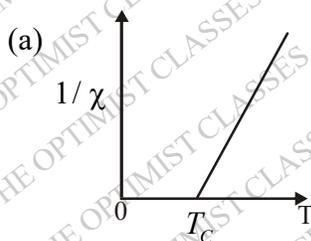
Q14. The coherence length of laser light is

- (a) directly proportional to the length of the active lasing medium
 (b) directly proportional to the width of the spectral line
 (c) inversely proportional to the width of the spectral line
 (d) inversely proportional to the length of the active lasing medium

Q15. Metallic monovalent sodium crystallizes in body centered cubic structure. If the length of the unit cell is $4 \times 10^{-8} \text{ cm}$, the concentration of conduction electrons in metallic sodium is

- (a) $6.022 \times 10^{23} \text{ cm}^{-3}$ (b) $3.125 \times 10^{22} \text{ cm}^{-3}$
 (c) $2.562 \times 10^{21} \text{ cm}^{-3}$ (d) $1.250 \times 10^{20} \text{ cm}^{-3}$

Q16. The plot of inverse magnetic susceptibility $1/\chi$ versus temperature T of an antiferromagnetic sample corresponds to



Q17. According to the quark model, the K^+ meson is composed which of the following quarks

- (a) uud (b) $u\bar{c}$ (c) $u\bar{s}$ (d) $s\bar{u}$

Q18. An ${}^8O^{16}$ nucleus is spherical and has a charge radius R and a volume $V = \frac{4}{3} \pi R^3$. According to empirical observations of the charge radii, the volume of the ${}_{54}Xe^{128}$ nucleus assumed to be spherical is

- (a) $8V$ (b) $2V$ (c) $6.75V$ (d) $1.89V$

Q19. A common emitter transistor amplifier circuit is operated under a fixed bias. In this circuit, the operating point.

- (a) remains fixed with an increase in temperature
 (b) moves towards cut-off region with a increase in temperature
 (c) moves towards the saturation region with a decrease in temperature
 (d) moves towards the saturation region with an increase in temperature

Q20. Under normal operating conditions, the gate terminal of an n -channel junction field effect transistor (JFET) and an n -channel metal oxide semiconductor field effect transistor (MOSFET) are

- (a) both biased with positive potentials
 (b) both biased with negative potentials

- (c) biased with positive and negative potentials, respectively
 (d) biased with negative and positive potentials, respectively

Q.21-Q.75 : Carry TWO marks each.

Q21. The eigenvalues of the matrix $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ are

- (a) $\frac{1}{2}(\sqrt{3} \pm i)$ when $\theta = 45^\circ$ (b) $\frac{1}{2}(\sqrt{3} \pm i)$ when $\theta = 30^\circ$
 (c) ± 1 since, the matrix is unitary (d) $\frac{1}{2}(1 \pm i)$ when $\theta = 30^\circ$

Q22. If the fourier transform $F[\delta(x-a)] = \exp(-i2\pi va)$, then $F^{-1}(\cos 2\pi va)$ will correspond to

- (a) $\delta(x-a) - \delta(x+a)$ (b) a constant
 (c) $\frac{1}{2}[\delta(x-a) + i\delta(x+a)]$ (d) $\frac{1}{2}[\delta(x-a) + \delta(x+a)]$

Q23. if $I = \oint_C \ln z dz$, Where C is the unit circle taken anticlockwise and $\ln z$ is the principal branch of the logarithm function, which of the following is correct?

- (a) $I = 0$ by residue theorem. (b) I is not defined since, $\ln z$ is branch cut.
 (c) $I \neq 0$ (d) $\oint_C \ln(z^2) dz = 2I$

Q24. The value of $\int_{-i}^i \pi(z+1) dz$ is

- (a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) $(-1+2i)\pi$

Q25. Consider the Bessel equation: ($\nu=0$), $\frac{d^2 y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + y = 0$

Which of the following statements is correct?

- (a) Equation has regular singular points at $z=0$ and $z=\infty$
 (b) Equation has 2 linearly independent solutions that are entire.
 (c) Equation has an entire solution and a second linearly independent solution is singular at $z=0$.
 (d) Limit $z \rightarrow \infty$ taken along X -axis, exists for both the linearly independent solution.

Q26. Under a certain rotation of coordinate axes, a rank-1 tensor v_a ($a=1,2,3$) transforms according to the or-

thogonal transformation defined by the relations $v_1 = \frac{1}{\sqrt{2}}(v_1 + v_2)$; $v_2 = \frac{1}{\sqrt{2}}(-v_1 + v_2)$; $v_3 = v_3$. Under the

same rotaion a rank-2 tensor $T_{a,b}$ would transform such that

- (a) $T'_{1,1} = T_{1,1} T_{1,2}$ (b) $T'_{1,1} = T_{1,1}$
 (c) $T'_{1,1} = T_{1,1} + 2T_{2,2} - T_{2,1}$ (d) $T'_{1,1} = \frac{1}{2}(T_{1,1} + T_{2,2} + T_{1,2} + T_{2,1})$

Q27. The Lagrangian of a system is given by $L = \frac{1}{2} \dot{q}^2 + q\dot{q} - \frac{1}{2} q^2$. It describes the motion of

- (a) A harmonic oscillator.
- (b) A damped harmonic oscillator.
- (c) An anharmonic oscillator
- (d) A system with unbounded motion.

Q28. The moment of inertia tensor of a rigid body is given by $I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$. The magnitude of the moment of

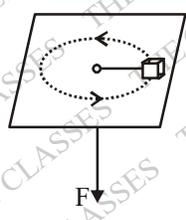
inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is

- (a) 6
- (b) 5
- (c) 2
- (d) 8/3

Q29. A hoop of radius R is pivoted at a point on the circumference. The period of small oscillations in the plane of the hoop is

- (a) $2\pi\sqrt{\frac{2R}{g}}$
- (b) $2\pi\sqrt{\frac{R}{4g}}$
- (c) $2\pi\sqrt{\frac{R}{g}}$
- (d) $2\pi\sqrt{\frac{9R}{7g}}$

Q30. A mass m is constrained to move on a horizontal frictionless surface. It is set in circular motion with radius r_0 and angular speed ω_0 by an applied force \vec{F} communicated through an inextensible thread that passes through a hole on the surface as shown in the figure.



This force is suddenly doubled. The magnitude of the radial velocity of the mass

- (a) increases till the mass falls into the hole
- (b) decreases till the mass falls into the hole
- (c) remains constant
- (d) becomes zero at a radius r_1 where $0 < r_1 < r_0$

Q31. For a simple harmonic oscillator the Lagrangian is given by $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$. If $A(p, q) = \frac{p+iq}{\sqrt{2}}$ and $H(p, q)$ is the Hamiltonian of the system, the Poisson bracket $\{A(p, q), H(p, q)\}$ is given by

- (a) $iA(p, q)$
- (b) $A^*(p, q)$
- (c) $-iA^*(p, q)$
- (d) $-iA(p, q)$

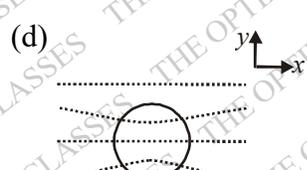
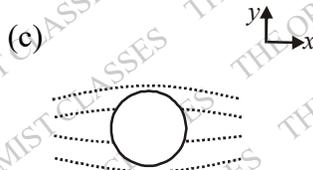
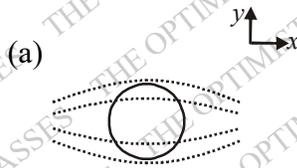
Q32. A plane electromagnetic wave is given by $E_0(\hat{x} + e^{i\delta}\hat{y})\exp\{i(kz - \omega t)\}$. At a given location, the number of times \vec{E} vanishes in one second is :

- (a) An integer near $\frac{\omega}{\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer
- (b) An integer near $\frac{\omega}{\pi}$ and is independent of δ

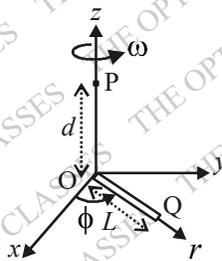
(c) An integer near $\frac{\omega}{2\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer

(d) An integer near $\frac{\omega}{2\pi}$ and is independent of δ

Q33. A dielectric sphere is placed in a uniform electric field directed along the positive y -axis. Which one of the following represents the correct equipotential surfaces?



Q34. A rod of length L with uniform charge density λ per unit length is in the xy -plane and rotating about z -axis passing through one of its edge with an angular velocity $\vec{\omega}$ as shown in the figure below.



$(\hat{r}, \hat{\phi}, \hat{z})$ refer to the unit vectors at Q , \vec{A} is the vector potential at a distance d from the origin O along z -axis for $d \gg L$ and \vec{J} is the current density due to the motion of the rod. Which one of the following statements is correct?

(a) \vec{J} along \hat{r} ; \vec{A} along \hat{z} ; $|\vec{A}| \propto \frac{1}{d}$

(b) \vec{J} along $\hat{\phi}$; \vec{A} along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d^2}$

(c) \vec{J} along \hat{r} ; \vec{A} along \hat{z} ; $|\vec{A}| \propto \frac{1}{d^2}$

(d) \vec{J} along $\hat{\phi}$; \vec{A} along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d}$

Q35. A circular disc of radius a on the xy plane has a surface charge density $\sigma = \frac{\sigma_0 r \cos \theta}{a}$. The electric dipole moment of this charge distribution is

(a) $\frac{\sigma_0 \pi a^4}{4} \hat{x}$

(b) $\frac{\sigma_0 \pi a^3}{4} \hat{x}$

(c) $-\frac{\sigma_0 \pi a^3}{4} \hat{x}$

(d) $-\frac{\sigma_0 \pi a^4}{4} \hat{x}$

Q36. At time $t=0$, a charge distribution $\rho(\vec{r}, 0)$ exists within an ideal homogeneous conductor of permittivity ϵ and conductivity σ . At a later time $\rho(\vec{r}, t)$ is given by

(a) $\rho(\vec{r}, t) = \rho(\vec{r}, 0) \exp\left(-\frac{\sigma t}{\epsilon}\right)$

(b) $\rho(\vec{r}, t) = \frac{\rho(\vec{r}, 0)}{1 + (\sigma t / \epsilon)^2}$

$$(c) \rho(\vec{r}, t) = \rho(\vec{r}, 0) \exp \left[-\left(\frac{\sigma t}{\varepsilon} \right)^2 \right] \quad (d) \rho(\vec{r}, t) = \rho(\vec{r}, 0) \exp \frac{\varepsilon}{\sigma t} \sin \left(\frac{\sigma t}{\varepsilon} \right)$$

Q37. A nonrelativistic charged particle moves along the positive x -axis with a constant positive acceleration $a\hat{x}$. The particle is at the origin at $t = 0$. Radiation is observed at $t = 0$ at a distant point $(0, d, 0)$ on the y -axis. Which one of the following statements is correct?

- (a) The radiation is unpolarized.
- (b) The radiation is plane polarized with polarization parallel to the x -axis
- (c) The radiation is plane polarized with polarization parallel to the xy plane along a line inclined to the x -axis
- (d) The radiation is elliptically polarized

Q38. For a physical system, two observables O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below

- (a) every eigenstate of O_1 must necessarily be an eigenstate O_2
- (b) every non-degenerate eigenstate of O_1 must necessarily be an eigenstate of O_2
- (c) when an observation of O_1 is carried out on an arbitrary state $|\psi\rangle$ of the physical system, a subsequent observation of O_2 leads to an unambiguous result.
- (d) Observation of O_1 and O_2 , carried out on an arbitrary state $|\psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observations are made.

Q39. An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x = x_0$.

[The SHO has energy levels E_n ($n = 0, 1, 2, \dots$) and associated normalized wave-function Ψ_0]. Subsequently, an exact measurement of energy E is made. Using the general notation $Pr(E = E')$ denoting the probability that a result E' is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

- (a) $Pr(E = E_0) = 0$
- (b) $Pr(E = E_n) = 1$ for some value of n .
- (c) $Pr(E = E_n) \propto \Psi_0(x)$
- (d) $Pr(E > E'') > 0$ for any E''

Q40. Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let

I denote the quantum number associated with the total angular momentum and let $\langle m \rangle$ denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B is Bohr magneton)?

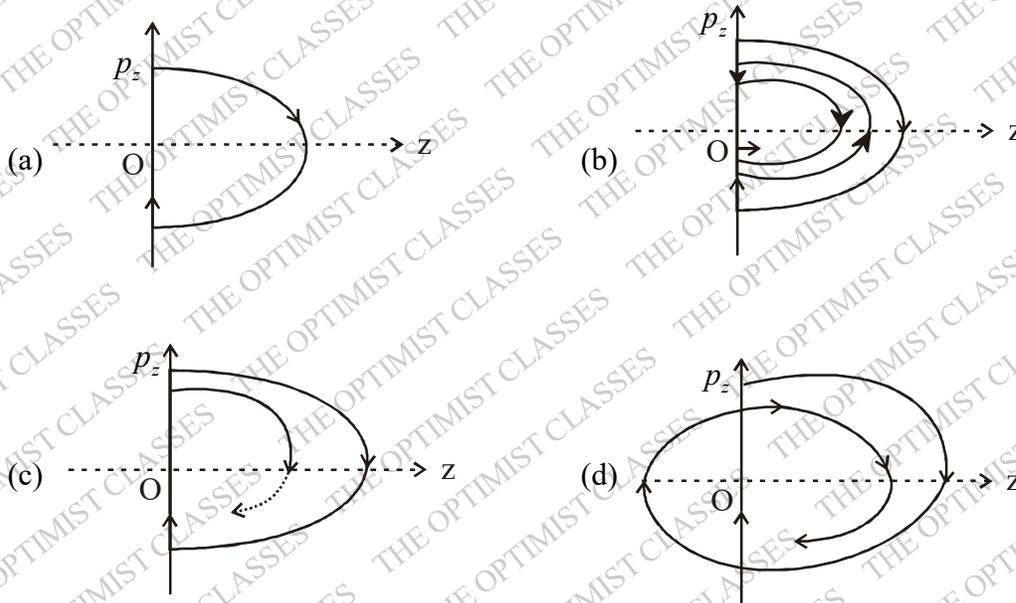
- (a) $I = 0, \langle m \rangle = 0$
- (b) $I = \frac{1}{2}, \langle m \rangle = 1\mu_B$
- (c) $I = 1, \langle m \rangle \approx 1\mu_B$
- (d) $I = 0, \langle m \rangle = 2\mu_B$

Q41. A particle is placed in a one dimensional box of size L along the x -axis ($0 < x < L$). Which of the following is true?

- (a) In the ground state, the probability of finding the particle in the interval $(L/4, 3L/4)$ is half.
- (b) In the first excited state, the probability of finding the particle in the interval $(L/4, 3L/4)$ is half. This also holds for states with $n = 4, 6, 8, \dots$
- (c) For an arbitrary state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half

(d) In the ground state, the particle has a definite momentum.

- Q42. An inelastic ball of mass m has been thrown vertically upwards from the ground at $z = 0$. The initial kinetic energy of the ball is E . The phase trajectory of the ball after successive bouncing on the ground is



- Q43. A system containing N non-interacting localized particles of spin $\frac{1}{2}$ and magnetic moment μ each is kept in constant external magnetic field B and in thermal equilibrium at temperature T . The magnetization of the system is,

(a) $N\mu \coth\left(\frac{\mu B}{k_B T}\right)$ (b) $N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$
 (c) $N\mu \sinh\left(\frac{\mu B}{k_B T}\right)$ (d) $N\mu \cosh\left(\frac{\mu B}{k_B T}\right)$

- Q44. Two identical particles have to be distributed among three energy levels. Let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given energy state. The subscripts B, E and C correspond to whether the particles are bosons, fermions and classical particles, respectively. The $r_B : r_F : r_C$ is equal to

(a) $\frac{1}{2} : 0 : 1$ (b) $1 : \frac{1}{2} : 1$ (c) $1 : \frac{1}{2} : \frac{1}{2}$ (d) $1 : 0 : \frac{1}{2}$

- Q45. A photon gas is at thermal equilibrium at temperature T . The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is:

(a) $\exp\left(\frac{\hbar\omega}{k_B T}\right) + 1$ (b) $\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1$ (c) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right) + 1\right)^{-1}$ (d) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1\right)^{-1}$

- Q46. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V . It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V . Once the combined system reaches equilibrium,

- (a) the total entropy of the final system is the same as the sum of the entropy of the individual system always.
 (b) the entropy of mixing is $2Nk_B \ln 2$.
 (c) the entropy of the final system is less than that of sum of the initial entropies of the two gases.

(d) the entropy of mixing is non-zero when the atoms A and B are of the same type.

Q47. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0, \varepsilon$ and 2ε having degeneracies $g(E) = 1, 2$ and 4 respectively. The mean energy of the system is:

(a) $\varepsilon \left(\frac{4 \exp(-\varepsilon/k_B T) + 8 \exp(-2\varepsilon/k_B T)}{1 + 2 \exp(-\varepsilon/k_B T) + 4 \exp(-2\varepsilon/k_B T)} \right)$ (b) $\varepsilon \left(\frac{2 \exp(-\varepsilon/k_B T) + 8 \exp(-2\varepsilon/k_B T)}{1 + 2 \exp(-\varepsilon/k_B T) + 4 \exp(-2\varepsilon/k_B T)} \right)$

(c) $\varepsilon \left(\frac{2 \exp(-\varepsilon/k_B T) + 4 \exp(-2\varepsilon/k_B T)}{1 + 2 \exp(-\varepsilon/k_B T) + 4 \exp(-2\varepsilon/k_B T)} \right)^2$ (d) $\varepsilon \left(\frac{\exp(-\varepsilon/k_B T) + 2 \exp(-2\varepsilon/k_B T)}{1 + \exp(-\varepsilon/k_B T) + \exp(-2\varepsilon/k_B T)} \right)$

Q48. Three consecutive absorption lines at 64.275 cm^{-1} , 77.130 cm^{-1} and 89.985 cm^{-1} have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia I_A and I_B are (I_A is with respect to the bond axis passing through the centre of mass and I_B is with respect to an axis passing through the centre of the mass and perpendicular to bond axis).

(a) both equal to $\frac{\hbar^2}{12.855hc} \text{ gm cm}^2$ (b) zero and $\frac{\hbar^2}{12.855hc} \text{ gm cm}^2$

(c) both equal to $\frac{\hbar^2}{6.427hc} \text{ gm cm}^2$ (d) zero and $\frac{\hbar^2}{6.427hc} \text{ gm cm}^2$

Q49. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency ν_e . The frequency of two consecutive Stokes lines are

(a) $\nu_e - 10B, \nu_e - 14B$ (b) $\nu_e - 2B, \nu_e - 4B$

(c) $\nu_e + 10B, \nu_e + 14B$ (d) $\nu_e + 2B, \nu_e + 4B$

Q50. Which of the following statement is **INCORRECT** in vibrational spectroscopy with anharmonicity?

- (a) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, \dots$
- (b) Anharmonicity leads to multiple absorption lines
- (c) The intensities of hot band lines are stronger than the fundamental absorption
- (d) The frequencies of hot band lines are smaller than the fundamental absorption

Q51. The molecular spectra of two linear molecules $O-C-O$ and $O-C-S$ are recorded in the microwave region. Which of the following statement is correct?

- (a) Both the molecules would show absorption lines
- (b) Both the molecules would not show absorption lines
- (c) $O-C-O$ would show absorption lines, but not $O-C-S$
- (d) $O-C-S$ would show absorption lines, but not $O-C-O$

Q52. When the refractive index μ of the active medium changes by $\Delta\mu$ in a laser resonator of length L , the changes in the spectral spacing between the longitudinal modes of the laser is (c is the speed of light in free space)

(a) $\frac{c}{2(\mu + \Delta\mu)L}$ (b) $\frac{c}{2\Delta\mu L}$ (c) $\frac{c}{2L} \left[\frac{\Delta\mu}{\mu(\mu + \Delta\mu)} \right]$ (d) zero

Q53. The primitive translation vectors of the body centered cubic lattice are

$$\vec{a} = \frac{a}{2}(\hat{x} + \hat{y} - \hat{z}), \quad \vec{b} = \frac{a}{2}(-\hat{x} + \hat{y} + \hat{z}) \text{ and } \vec{c} = \frac{a}{2}(\hat{x} - \hat{y} + \hat{z})$$

The primitive translation vectors \vec{A}, \vec{B} and \vec{C} of the reciprocal lattice are

$$(a) \vec{A} = \frac{2\pi}{a}(\hat{x} - \hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y} + \hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x} + \hat{z})$$

$$(b) \vec{A} = \frac{2\pi}{a}(\hat{x} + \hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y} - \hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x} + \hat{z})$$

$$(c) \vec{A} = \frac{2\pi}{a}(\hat{x} + \hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y} + \hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x} - \hat{z})$$

$$(d) \vec{A} = \frac{2\pi}{a}(\hat{x} + \hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y} + \hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x} + \hat{z})$$

Q54. The structure factor of a single cell of identical atoms of form factor f is given by

$$S_{hkl} = f \sum_j \exp(-i2\pi(x_j h + y_j k + z_j l)) \text{ where } (x_j, y_j, z_j) \text{ is the coordinate of an atom, and } hkl \text{ are the}$$

Miller indices. Which one of the following statement is correct for the diffraction peaks of body centered cubic (BCC) and face centered cubic (FCC) lattices?

$$(a) \text{ BCC : } (200); (110); (220)$$

$$(b) \text{ BCC : } (210); (110); (222)$$

$$\text{FCC : } (111); (311); (400)$$

$$\text{FCC : } (111); (311); (400)$$

$$(c) \text{ BCC : } (200); (110); (222)$$

$$(d) \text{ BCC : } (200); (210); (222)$$

$$\text{FCC : } (111); (211); (400)$$

$$\text{FCC : } (111); (211); (400)$$

Q55. The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model, Einstein model and Debye model. At low temperature $\hbar\omega \gg k_B T$, which one of the following statements is true (a and A are constants)

$$(a) \text{ Dulong Petit : } C \propto \exp(-a/T); \text{ Einstein : } C = \text{Constant}; \text{ Debye: } C \propto \left(\frac{T}{A}\right)^3$$

$$(b) \text{ Dulong Petit : } C = \text{constant}; \text{ Einstein : } C \propto \left(\frac{T}{A}\right)^3; \text{ Debye: } C \propto \exp(-a/T)$$

$$(c) \text{ Dulong Petit : } C = \text{constant}; \text{ Einstein : } C \propto \frac{e^{-a/T}}{T^2}; \text{ Debye: } C \propto \left(\frac{T}{A}\right)^3$$

$$(d) \text{ Dulong Petit : } C \propto \left(\frac{T}{A}\right)^3; \text{ Einstein : } C \propto \frac{e^{-a/T}}{T^2}; \text{ Debye: } C = \text{constant}$$

Q56. A linear diatomic lattice of lattice constant a with masses M and m ($M > m$) are coupled by a force constant C . The dispersion relation is given by

$$\omega_{\pm}^2 = C \left(\frac{M+m}{Mm} \right) \pm \left[C^2 \left(\frac{M+m}{Mm} \right)^2 - \frac{4C^2}{Mm} \sin^2 \frac{ka}{2} \right]^{1/2}$$

Which one of the following statements is **INCORRECT**?

(a) The atoms vibrating in transverse mode correspond to the optical branch

(b) The maximum frequency of the acoustic branch depends on the mass of the lighter atom m .

(c) The dispersion of frequency in the optical branch is smaller than that in the acoustic branch

(d) No normal modes exist in the acoustic branch for any frequency greater than the maximum frequency at

$$k = \pi/a$$

Q57. The kinetic energy of a free electron at a corner of the first Brillouin zone of a two dimensional square lattice is larger than that of an electron at the mid-point of a side of the zone by a factor b . The value of b is

$$(a) b = \sqrt{2}$$

$$(b) b = 2$$

$$(c) b = 4$$

$$(d) b = 8$$

Q58. An intrinsic semiconductor with mass of a hole m_h and mass of an electron m_e is at a finite temperature T . If the top of the valence band energy is E_v and the bottom of the conduction band energy is E_c , the Fermi energy of the semiconductor is

$$(a) E_F = \left(\frac{E_v + E_c}{2} \right) - \frac{3}{4} k_B T \ln \left(\frac{m_h}{m_e} \right) \quad (b) E_F = \left(\frac{k_B T}{2} \right) + \frac{3}{4} (E_v + E_c) \ln \left(\frac{m_h}{m_e} \right)$$

$$(c) E_F = \left(\frac{E_v + E_c}{2} \right) + \frac{3}{4} k_B T \ln \left(\frac{m_h}{m_e} \right) \quad (d) E_F = \left(\frac{k_B T}{2} \right) - \frac{3}{4} (E_v + E_c) \ln \left(\frac{m_h}{m_e} \right)$$

Q59. Choose the correct statement from the following :

- (a) The reaction $K^+ K^- \rightarrow p \bar{p}$ can proceed irrespective of the kinetic energies of K^+ and K^-
 (b) The reaction $K^+ K^- \rightarrow p \bar{p}$ is forbidden by the Baryon number conservation.
 (c) the reaction $K^+ K^- \rightarrow 2\gamma$ is forbidden by strangeness conservation.
 (d) The decay $K^0 \rightarrow \pi^+ \pi^-$ proceeds via weak interaction.

Q60. The following gives a list of pairs containing

- (i) a nucleus (ii) one of its properties

Find the pair which is in appropriate.

- (a) (i) ${}_{10}\text{Ne}^{20}$ nucleus, (ii) stable nucleus;
 (b) (i) a spheroidal nucleus, (ii) an electric quadrupole moment;
 (c) (i) ${}_8\text{O}^{16}$ nucleus, (ii) nucleus spin $J = \frac{1}{2}$;
 (d) (i) U^{238} nucleus (ii) Binding energy = 1785 MeV approximately

Q61. The four possible configurations of neutrons in the ground state of ${}_4\text{Be}^9$ nucleus according to the shell model, and the associated nuclear spin are listed below. Choose the correct one.

$$(a) \left({}^1s_{1/2} \right)^2 \left({}^1p_{3/2} \right)^3; J = \frac{3}{2} \quad (b) \left({}^1s_{1/2} \right)^2 \left({}^1p_{1/2} \right)^2 \left({}^1p_{3/2} \right)^1; J = \frac{3}{2}$$

$$(c) \left({}^1s_{1/2} \right)^2 \left({}^1p_{3/2} \right)^4; J = \frac{1}{2} \quad (d) \left({}^1s_{1/2} \right)^2 \left({}^1p_{3/2} \right)^2 \left({}^1p_{1/2} \right)^1; J = \frac{1}{2}$$

Q62. The mass difference between the pair of mirror nuclei ${}_6\text{C}^{11}$ and ${}_5\text{B}^{11}$ is given to be $\Delta \text{MeV} / c^2$. According to the semi empirical mass formula, the mass difference between the pair of mirror nuclei ${}_{17}\text{F}$ and ${}_{17}\text{O}$ will approximately be (rest mass of proton $m_p = 938.27 \text{ MeV}/c^2$ and rest mass of neutron $m_n = 939.57 \text{ MeV}/c^2$).

- (a) $1.39\Delta \text{MeV} / c^2$ (b) $(1.39\Delta + 0.5) \text{ MeV} / c^2$
 (c) $0.86\Delta \text{MeV} / c^2$ (d) $(1.6\Delta + 0.78) \text{ MeV} / c^2$

Q63. A heavy nucleus is found to contain more neutrons than protons. This fact is related to which one of the following statements

- (a) The nuclear force between neutrons is stronger than that between protons.
 (b) The nuclear force between protons is of a shorter range than those between neutrons, so that a smaller number of protons are held together by the nuclear force.
 (c) Protons are unstable, to their number in a nucleus diminishes.

(d) It costs more energy to add a proton to a (heavy) nucleus, than a neutron because of the Coulomb repulsion between protons.

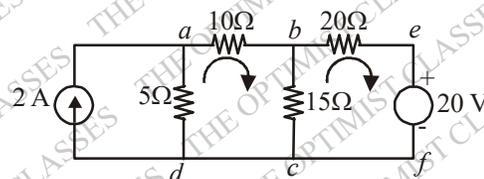
Q64. A neutral pi meson (π^0) has rest-mass of approximately $140 \text{ MeV}/c^2$ and a lifetime of τ sec. A π^0 produced in the laboratory is found to decay after 1.25τ sec into two photons. Which of the following sets represents a possible set of energies of the two photons as seen in the laboratory?

- (a) 70 MeV and 70 MeV (b) 35 MeV and 100 MeV
 (c) 75 MeV and 100 MeV (d) 25 MeV and 150 MeV

Q65. An a.c. voltage of $220 V_{rms}$ is applied to the primary of a 10:1 step-down transformer. The secondary of the transformer is centre tapped and connected to a full wave rectifier with a load resistance. The d.c. voltage appearing across the load is

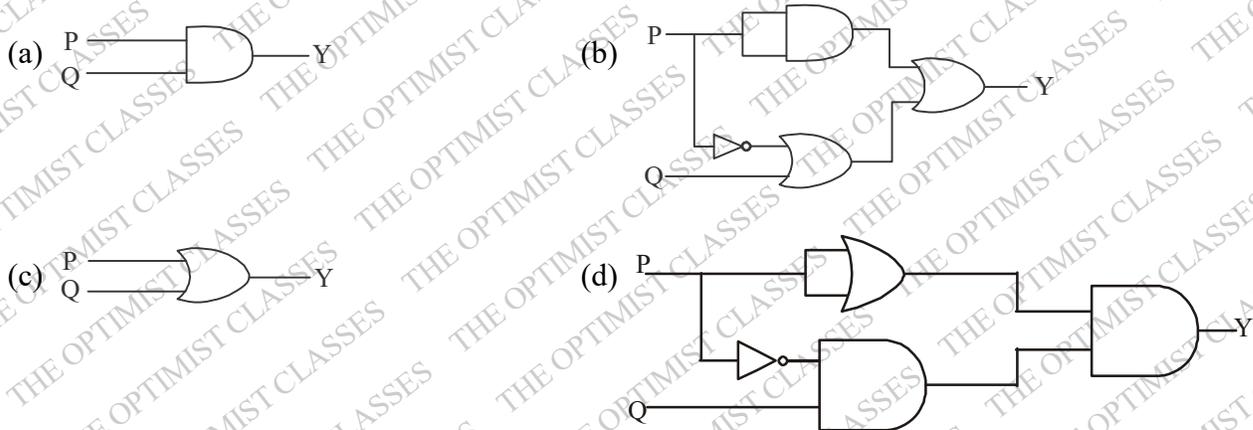
- (a) $\frac{22}{\pi}$ (b) $\frac{31}{\pi}$ (c) $\frac{62}{\pi}$ (d) $\frac{44}{\pi}$

Q66. Let I_1 and I_2 represent mesh currents in the loop $abcd$ and $befcb$ respectively. The correct expression describing Kirchoff's voltage lopp law in one of the following loops is



- (a) $30I_1 - 15I_2 = 10$ (b) $-15I_1 + 20I_2 = -20$
 (c) $30I_1 - 15I_2 = -10$ (d) $-15I_1 + 20I_2 = 20$

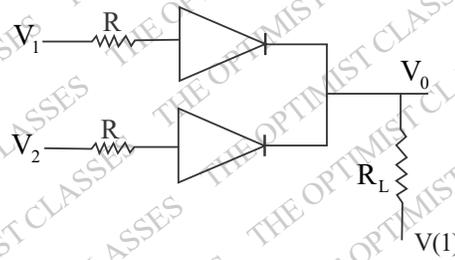
Q67. The simplest logic gate circuit corresponding to the Boolean expression, $Y = P + \bar{P}Q$ is :



Q68. An analog voltage V is converted into 2-bit binary number. The minimum number of comparators required and their reference voltages are

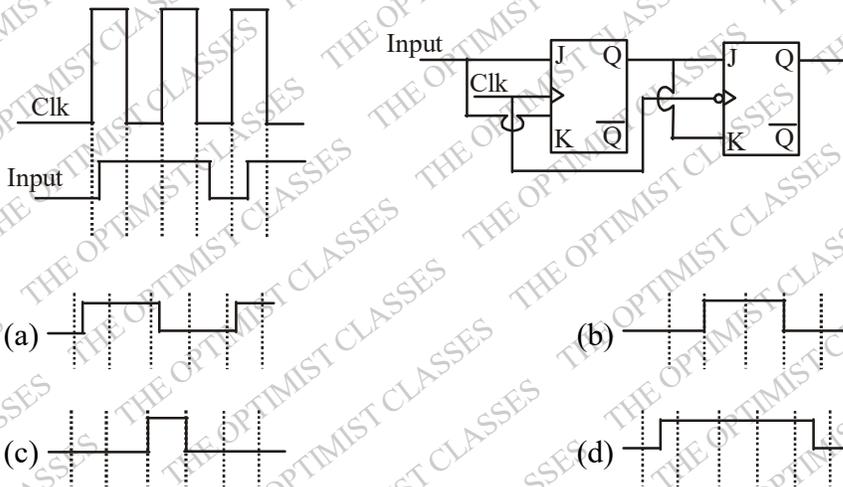
- (a) 3, $\left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}\right)$ (b) 3, $\left(\frac{V}{3}, \frac{2V}{3}, V\right)$
 (c) 4, $\left(\frac{V}{5}, \frac{2V}{5}, \frac{3V}{5}, \frac{4V}{5}\right)$ (d) 4, $\left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}, V\right)$

Q69. The following circuit (where $R_L \gg R$) performs the operation of



- (a) OR gate for a negative logic system
- (b) NAND gate for a negative logic system.
- (c) AND gate for a positive logic system.
- (d) AND gate for a negative logic system

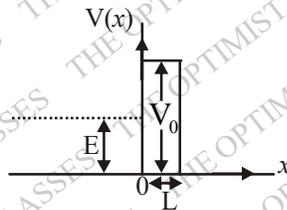
Q70. In the T type master-slave JK flip flop is shown along with the clock and input waveforms. The Q_n output of flip flop was zero initially. Identify the correct output waveform.



Common Data Questions

Common Data for Questions 71, 72 and 73 :

A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0 < x < L$) and height V_0 as shown in the figure ($E < V_0$)



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0 , A_r and A_t denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

Q71. Throughout $0 < x < L$, the wave-function

- (a) can be chosen to be real
- (b) is exponentially decaying
- (c) is generally complex
- (d) is zero

Q72. Let the probability current associated with the incident wave be S_0 . Let R be the reflection coefficient. Then

- (a) the probability current vanishes in the classically forbidden region
- (b) the probability current is TS_0 for $x > L$
- (c) for, $x < 0$ the probability current is $S_0(1 + R)$
- (d) for $x > L$, the probability current is complex

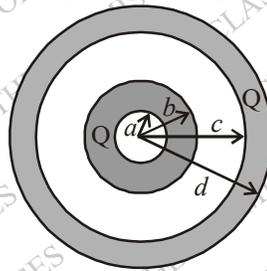
Q73. The ratio of the reflected to the incident amplitude A_B / A_0 is

- (a) $1 - A_r / A_0$
- (b) $\sqrt{1 - T}$ in magnitude

- (c) a real neagative number
- (d) $\sqrt{\left(1 - \left|\frac{A_r}{A_0}\right|^2\right) \frac{E}{V_0 - E}}$

Common Data for Questions 74 and 75 :

Consider two concentric conducting spherical shells with inner and outer radii a, b and c, d as shown in the figure. Both the shells are given Q amount of positive charges.



Q74. The electric field in different regions are :

(a) $\vec{E} = 0$ for $r < a$; $\vec{E} = \frac{-Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $a < r < b$; $\vec{E} = 0$ for $b < r < c$; $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $r > d$

(b) $\vec{E} = \frac{-Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $r < a$; $\vec{E} = 0$ for $a < r < b$; $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $b < r < c$; $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $r > d$

(c) $\vec{E} = \frac{-Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $r < a$; $\vec{E} = 0$ for $a < r < b$; $\vec{E} = 0$ for $b < r < c$; $\vec{E} = \frac{2Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $r > d$

(d) $\vec{E} = 0$ for $r < a$; $\vec{E} = 0$ for $a < r < b$; $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $b < r < c$; $\vec{E} = \frac{2Q}{4\pi\epsilon_0 r^2} \hat{r}$ for $r > d$

Q75. In order to have equal surface charge densities on the outer surfaces of both the shells, the following conditions should be satisfied

(a) $d = 4b$ and $c = 2a$

(b) $d = 2b$ and $c = \sqrt{2}a$

(c) $d = \sqrt{2}b$ and $c > a$

(d) $d > b$ and $c = \sqrt{2}a$

Linked Answer Questions : Q.76 to Q. 85 carry two marks each.

Statement for Linked Answer Questions 76 and 77

Consider the β -decay of a free neutron at rest in the laboratory.

- Q76. Which of the following configurations of the decay products correspond to the largest energy of the antineutrino $\bar{\nu}$? (rest mass of electron, $m_e = 0.51 \text{ MeV}/c^2$, rest mass of proton $m_p = 938.27 \text{ MeV}/c^2$ and rest mass of neutron $m_n = 939.57 \text{ MeV}/c^2$)
- In the laboratory, proton is produced at rest
 - In the laboratory, momenta of proton, electron and the anti-neutrino all have the same magnitude.
 - In the laboratory, proton and electron fly-off with (nearly) equal and opposite momenta.
 - In the laboratory, electron is produced at rest.
- Q77. Using the result of above problem, answer the following. Which of the following represents approximately the maximum allowed energy of the anti-neutrino $\bar{\nu}$?
- 1.3 MeV
 - 0.8 MeV
 - 0.5 MeV
 - 2.0 MeV

Statement for Linked Answer Questions 78 and 79:

Consider two dimensional electron gas of N electrons of mass ' m ' each in a system of size $L \times L$.

- Q78. The density of states between energy ε and $\varepsilon + d\varepsilon$ is
- $\frac{4\pi L^2 m}{h^2} d\varepsilon$
 - $\frac{4\pi L^2 m}{h^2} \frac{1}{\sqrt{\varepsilon}} d\varepsilon$
 - $\frac{4\pi L^2 m}{h^2} \sqrt{\varepsilon} d\varepsilon$
 - $\frac{4\pi L^2 m}{h^2} \varepsilon d\varepsilon$
- Q79. The ground state energy E_0 of the system in terms of the Fermi energy E_f and the number of electrons N is given by
- $\frac{1}{3} NE_f$
 - $\frac{1}{2} NE_f$
 - $\frac{2}{3} NE_f$
 - $\frac{3}{5} NE_f$

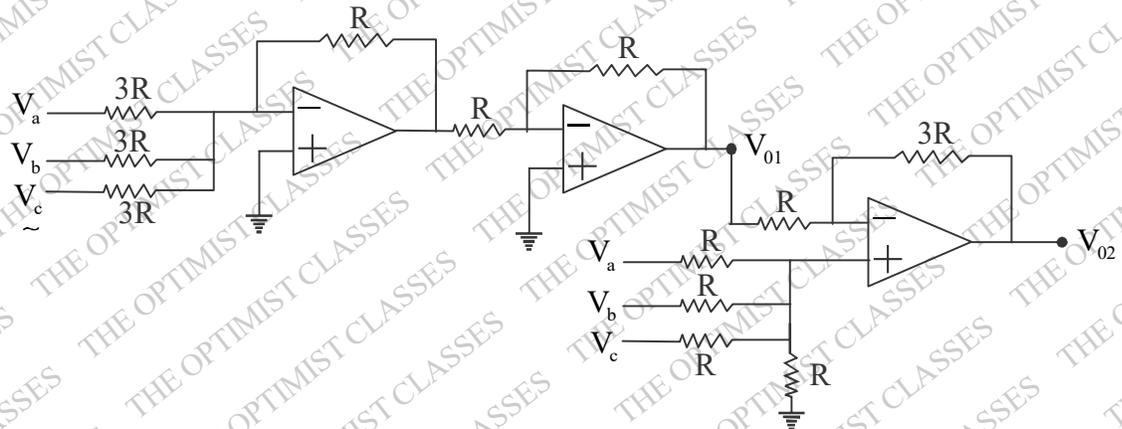
Statement for Linked Answer Questions 80 and 81 :

The rate of a clock in a spaceship "Suryashakti" is observed from earth to be $3/5$ of the rate of the clocks on earth.

- Q80. The speed of the spaceship "Suryashakti" relative to earth is
- $\frac{4}{5} c$
 - $\frac{3}{5} c$
 - $\frac{9}{10} c$
 - $\frac{2}{5} c$
- Q81. The rate of a clock in a spaceship "Aakashganga" is observed from earth to be $5/13$ of the rate of the clocks on earth. If both Aakashganga and Suryashakti are moving in the same direction relative to someone on earth, then the speed of Aakashganga relative to Suryashakti is
- $\frac{12}{13} c$
 - $\frac{4}{5} c$
 - $\frac{8}{17} c$
 - $\frac{5}{6} c$

Statement for Linked Answer Questions 82 and 83:

The following circuit contains three operational amplifiers and resistors.



Q82. The output voltage at the end of second operational amplifier V_{01} is:

(a) $V_{01} = 3(V_a + V_b + V_c)$

(b) $V_{01} = -\frac{1}{3}(V_a + V_b + V_c)$

(c) $V_{01} = \frac{1}{3}(V_a + V_b + V_c)$

(d) $V_{01} = \frac{4}{3}(V_a + V_b + V_c)$

Q83. The output V_{02} (at the end of third OP-AMP) of the above circuit is:

(a) $V_{02} = 2(V_a + V_b + V_c)$

(b) $V_{02} = 3(V_a + V_b + V_c)$

(c) $V_{02} = -\frac{1}{2}(V_a + V_b + V_c)$

(d) Zero

Statement for Linked Answer Questions 84 and 85 :

The set V for all polynomials of a real variable x of degree two or less and with real coefficients, constitutes a real linear vector space $V \equiv \{c_0 + c_1x + c_2x^2 : c_0, c_1, c_2 \in R\}$.

Q84. For $f(x) = a_0 + a_1x + a_2x^2 \in V$ and $g(x) = b_0 + b_1x + b_2x^2 \in V$, which one of the following constitutes an acceptable scalar product?

(a) $(f, g) = a_0^2b_0 + a_1^2b_1 + a_2^2b_2$

(b) $(f, g) = a_0^2b_0^2 + a_1^2b_1^2 + a_2^2b_2^2$

(c) $(f, g) = a_0b_0 - a_1b_1 + a_2b_2$

(d) $(f, g) = a_0b_0 - \frac{a_1b_1}{2} + \frac{a_2b_2}{3}$

Q85. Using the scalar product obtained in the above questions, identify the subspace of V that is orthogonal to $(1+x)$:

(a) $\{f(x) : b(1-x) + cx^2; b, c \in R\}$

(b) $\{f(x) : b(1-2x) + cx^2; b, c \in R\}$

(c) $\{f(x) : b + cx^2; b, c \in R\}$

(d) $\{f(x) : bx + cx^2; b, c \in R\}$

ANSWER KEY

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (d) | 3. (b) | 4. (c) | 5. (b) |
| 6. (c) | 7. (c) | 8. (a) | 9. (a) | 10. (b) |
| 11. (d) | 12. (a) | 13. (b) | 14. (c) | 15. (b) |
| 16. (b) | 17. (c) | 18. (a) | 19. (d) | 20. (b) |
| 21. (b) | 22. (d) | 23. (a) | 24. (b) | 25. (c) |
| 26. (d) | 27. (a) | 28. (b) | 29. (a) | 30. (d) |
| 31. (a) | 32. (a) | 33. (d) | 34. (b) | 35. (b) |
| 36. (a) | 37. (b) | 38. (d) | 39. (*) | 40. (b) |
| 41. (c) | 42. (c) | 43. (b) | 44. (d) | 45. (d) |
| 46. (b) | 47. (b) | 48. (b) | 49. (a) | 50. (c) |
| 51. (d) | 52. (c) | 53. (d) | 54. (a) | 55. (c) |
| 56. (b) | 57. (b) | 58. (c) | 59. (d) | 60. (c) |
| 61. (a) | 62. (b) | 63. (d) | 64. (c) | 65. (d) |
| 66. (a) | 67. (c) | 68. (a) | 69. (d) | 70. (*) |
| 71. (a) | 72. (b) | 73. (b) | 74. (d) | 75. (c) |
| 76. (d) | 77. (b) | 78. (a) | 79. (b) | 80. (a) |
| 81. (c) | 82. (c) | 83. (d) | 84. (c) | 85. (a) |