THE OPTIMIST CLASSES
IIT-JAM TOPPERS

8 AIR

MANOJ KUMAR SINGH

9 AIR

ABHAY

12 AIR

PAWAN

15 AIR

SATYAM

21 AIR

SOUMIL GIRISH SAHU

29 AIR

BHOOMIJA

31 AIR

AKSHIT AGGARWAL

32 AIR

SHIKHAR CHAMOLI

33 AIR

RAVI SINGH ADHIKARI

44 AIR

GAURAV JHA

56 AIR

SWAPNIL JOSHI

65 AIR

LOKESH BHATT

75 AIR

GOPESH VISHVAKARMA

77 AIR

VAIBHAV

80 AIR

SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022

ANNU
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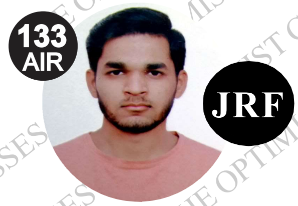


ALANKAR
UP15000162

SAHIL RANA
HR09000108

JAYESTHI
RJ11000161

DASRATH
RJ06000682

VIVEK
UK01000439

UZAIR AHMED
UP02000246

SURYA PRATAP SINGH
RJ06000232

HIMANSHU
UP10000095

CHANDAN
RJ09000159

SAIKHOM JOHNSON
MN01000196

AJAY SAINI
RJ06001744

VIKAS YADAV
RJ06001102

JYOTSNA KOHLI
UK02000262

SHYAM SUNDAR
RJ06000615

GATE PAPER 2009

Q.1-Q20 : Carry ONE mark each.

Q1. The value of the contour integral $\left|\int_C \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with center at the origin is

(a) $2\pi r$ (b) $\frac{r^2}{2}$ (c) πr^2 (d) r

Q2. An electrostatic field $\vec{E}$ exists in a given region R. Choose the WRONG statement.

(a) Circulation of $\vec{E}$ is zero

(b) $\vec{E}$ can always be expressed as the gradient of a scalar field

(c) The potential difference between any two arbitrary points in the region R is zero

(d) The work done in a closed path lying entirely in R is zero

Q3. The Lagrangian of a free particle in spherical polar co-ordinates is given by $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta\right)$.

The quantity that conserved is

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$ (c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}} + \dot{r}\dot{\theta}$

Q4. A conducting loop L of surface area S is moving with a velocity $\vec{v}$ in a magnetic filed $\vec{B}(\vec{r},t) = \vec{B}_0 t^2$, B_0 is a positive constant of suitable dimensions. The emf induced, V_{emf}, in the loop is given by

(a) $-\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$

(b) $\oint_L \left(\vec{v} \times \vec{B}\right).d\vec{L}$

(c) $-\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S} - \oint_L \left(\vec{v} \times \vec{B}\right).d\vec{L}$

(d) $-\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S} + \oint_L \left(\vec{v} \times \vec{B}\right).d\vec{L}$

Q5. The eigenvalues of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are

(a) real and distinct

(b) complex and distinct

(c) complex and coinciding

(d) real and coinciding

Q6. $\sigma_i\,(i=1,2,3)$ represent the Pauli spin matrices. Which of the following is NOT true?

(a) $\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}$ (b) $Tr(\sigma_i) = 0$

(c) The eigenvalues of σ_i are ± 1 (d) det $(\sigma_i) = 1$

Q7. Which one of the function given below represents the bound state eigen function of the operator $-\frac{d^2}{dx^2}$ in the region $0 \le x < \infty$, with the eigenvalue -4?

(a) $A_0 e^{2x}$ (b) $A_0 \cosh 2x$ (c) $A_0 e^{-2x}$ (d) $A_0 \sinh 2x$

Q8. Pick the wrong statement

(a) the nuclear force is independent of electric charge

(b) The Yukawa potential is proportinoal to $r^{-1}\exp\left(\frac{mc}{h}r\right)$. Where, r is the seperation between two nucleons.

(c) The range of nuclear force is of the order of $10^{-15} - 10^{-14}\,m$

(d) the nucleons interact among each other by the exchange of mesons.

Q9. If p and q are the position and momentum variables, which one of the following is NOT a canonical transformation?

(a) $Q = \alpha q$ and $P = \frac{1}{\alpha}p,\ for\ \alpha \neq 0$

(b) $Q = \alpha q + \beta p$ and $P = \beta q + \alpha p$ for α, β real and $\alpha^2 - \beta^2 = 1$

(c) $Q = p$ and $P = q$

(d) $Q = p$ and $P = -q$

Q10. The common Mode Rejection Ratio (CMRR) of a differential amplifier using an operational amplifier is $100dB$. The output voltage for a differential input of $200\,\mu V$ is $2V$. The common mode gain is

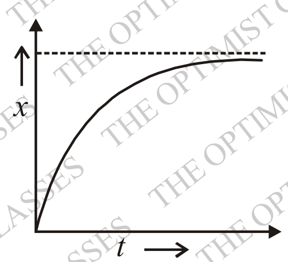
(a) 10 (b) 0.1 (c) 30 dB (d) 10 dB

Q11. In an insulating solid which one of the following physical phenomena is a consequence of Pauli's exclusion priciple?

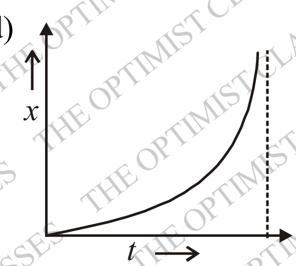
(a) Ionic conduactivity (b) Ferromagnetism (c) Paramagnetism (d) Ferroelectricity

Q12. Which one of the following curves gives the solution of the differential equation $k_1\frac{dx}{dt} + k_2 x = k_3$, where k_1, k_2 and k_3 are positive real constants with initial conditions $x = 0$ at $t = 0$?

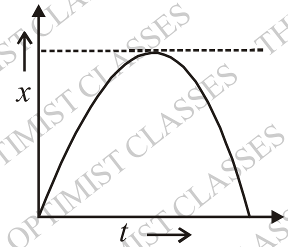
(a)



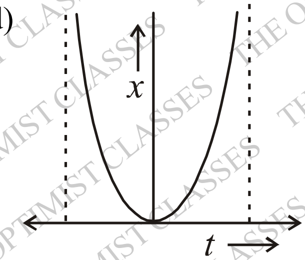
(d)



(c)



(d)



Q13. Identify which one is a first order phase transition?

(a) A liquid to gas transition at its critical temperature.

(b) A liquid to gas transition close to its triple point.

(c) A paramegnetic to ferromagnetic transition in the absence of a magnetic field.

(d) A metal to superconductor transition in the absence of a magnetic field.

Q14. **Group -I** lists some physical phenomena while **Group - II** gives some physical parameters. Match the phenomena with the corresponding parameter.

Group I	**Group II**
P. Doppler Broadening	1. Moment of inertia
Q. Natural Broadening	2. Refractive index
R. Rotational spectrum	3. Lifetime of the energy level
S. Total internal reflection	4. Pressure

(a) P–4, Q-3, R–1, S–2 (b) P–3, Q–2, R–1, S–4

(c) P–2, Q–3, R–4, S–1 (d) P–1, Q–4, R–2, S–3

Q15. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant, B is :

(a) $2B$ (b) $4B$ (c) $6B$ (d) $12B$

Q16. A superconducting ring is cooled in the presence of a magnetic field below its critical temperature (T_C). The total magnetic flux that passes through the ring is

(a) zero (b) $n\frac{h}{2e}$ (c) $\frac{nh}{4\pi e}$ (d) $\frac{ne^2}{hc}$

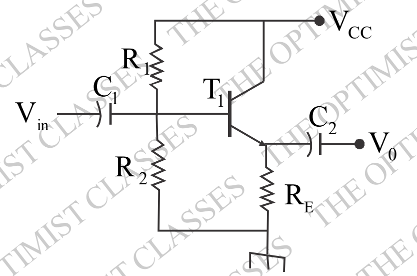
Q17. In a cubic crystal, atoms of mass M_1 lie on one set of planes and atoms of mass M_2 lie on planes inter leaved between those of the first set. If C is the force constant between nearest neighbour planes, the frequency of lattice vibrations for the optical phonon branch with wavevector $k = 0$ is

(a) $\sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (b) $\sqrt{C\left(\frac{1}{2M_1}+\frac{1}{M_2}\right)}$ (c) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{2M_2}\right)}$ (d) zero

Q18. In the quark model which one of the following represents a proton ?

(a) udd (b) uud (c) $u\bar{b}$ (d) $c\bar{c}$

Q19. The circuit shown below



(a) Is a common-emitter amplifier (b) Uses a pnp transistor

(c) Is an oscillator (d) Has a voltage gain less than one.

Q20. Conside a nucleus with N neutrons and Z protons. If m_p, m_n and $B.E.$ represents the mass of the proton, the mass of the neutron and the binding energy of the nucleus respectively and c is the velocity of light in free space, the mass of the nucleus is given by

(a) $Nm_n + Zm_p$ (b) $Nm_p + Zm_n$ (c) $Nm_n + Zm_p - \frac{B.E.}{c^2}$ (d) $Nm_p + Zm_n + \frac{B.E.}{c^2}$

Q.21 - Q.60 : Carry TWO marks each

Q21. The magnetic field (in $A\,m^{-1}$) inside a long solid cylindrical conductor of radius $a = 0.1\,m$ is,

$\vec{H} = \frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(ar)\right]\hat{\phi}$, where $\alpha = \frac{\pi}{2a}$ What is the total current (in A) in the conductor ?

(a) $\frac{\pi}{2a}$ (b) $\frac{800}{\pi}$ (c) $\frac{400}{\pi}$ (d) $\frac{300}{\pi}$

Q22. Which one of the following current densities, $\vec{J}$, can generate the magnetic vector potential $\vec{A} = \left(y^2\hat{i} + x^2\hat{j}\right)$?

(a) $\frac{2}{\mu_0}\left(x\hat{i} + y\hat{j}\right)$ (b) $-\frac{2}{\mu_0}\left(\hat{i} + \hat{j}\right)$ (c) $\frac{2}{\mu_0}\left(\hat{i} - \hat{j}\right)$ (d) $\frac{2}{\mu_0}\left(x\hat{i} - y\hat{j}\right)$

Q23. The value of the integral $\int_C \frac{e^z}{z^2 - 3z + 2}dz$, where the contour C is the circle $|z| = \frac{3}{2}$ is

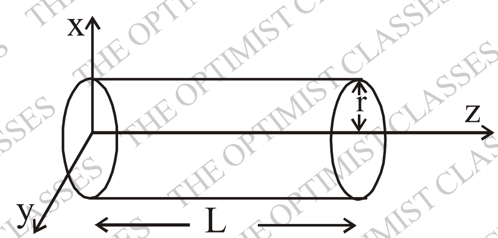
(a) $2\pi ie$ (b) πie (c) $-2\pi ie$ (d) $-\pi ie$

Q24. In a non-conducting medium characterized by $\varepsilon = \varepsilon_0, \mu = \mu_0$ and conductivity $\sigma = 0$, the electric field $\left(\text{in } Vm^{-1}\right)$ is given by $\vec{E} = 20\sin\left[10^8 t - kz\right]\hat{j}$. The magnetic field, $\vec{H}\left(\text{in } A\,m^{-1}\right)$, is given by :

(a) $20k\cos\left[10^8 t - kz\right]\hat{i}$ (b) $\frac{20k}{10^8\mu_0}\sin\left[10^8 t - kz\right]\hat{j}$

(c) $-\frac{20k}{10^8\mu_0}\sin\left[10^8 t - kz\right]\hat{i}$ (d) $-20k\cos\left[10^8 t - kz\right]\hat{j}$

Q25. A cylindrical rod of length L and radius r, made of an inhomogeneous dielectric, is placed with its axis along the z direction with one end at the origin as shown below:



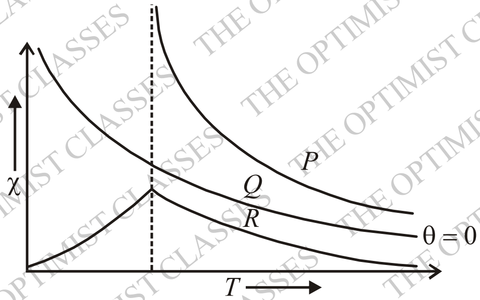
If the rod carries a polarization, $\vec{P} = \left(5z^2 + 7\right)\hat{k}$, the volume bound charge inside the dielectric is :

(a) zero (b) $10\pi r^2 L$ (c) $-5\pi r^2 L$ (d) $-5\pi r^2 L^2$

Q26. Let $T_{ij} = \sum_k \varepsilon_{ijk} a_k$ and $\beta_k = \sum_{i,j} \varepsilon_{ijk} T_{ij}$ where ε_{ijk} is the Levi-Civita density, defined to be zero if two of the indices coincide and +1 and -1 depending on whether ijk is even or odd permutation of 1,2,3. Then β_3 is equal to

(a) $2a_3$ (b) $-2a_3$ (c) a_3 (d) $-a_3$

Q27. The dependece of the magnetic susceptibility (χ) of a material with temperature (T) can be represented by $\chi \propto \frac{1}{T-\theta}$, where θ is the Curie-Weiss temperature. The plot of magnetic susceptibility versus temperature is sketched in the figure, as curves P, Q and R with curve Q having $\theta = 0$. Which one of the following statements is correct?



(a) Curve R represents a paramagnet and Q a ferromagnet
(b) Curve Q represents a ferromagnet and P an antiferromagnetic
(c) Curve R represents an antiferromagnet and Q a paramagnetic
(d) Curve R represents an antiferromagnet and Q a ferromagnet

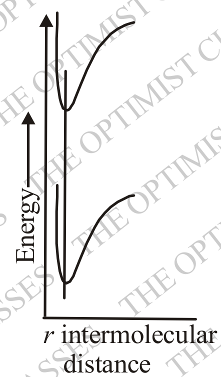
Q28. The dielectric constant of a material at optical frequencies is mainly due to
(a) inonic polarizability (b) electronic polarizability
(c) dipolar polarizability (d) ionic and dipolar polarizability

Q29. An electron of wavevector $\vec{k}_e$, velocity $\vec{v}_e$ and effective mass m_e is removed from a filled energy band. The resulting hole has wavevector $\vec{k}_h$, velocity $\vec{v}_h$, and effective mass m_h. Which one of the following statements is correct?

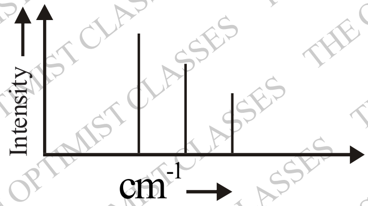
(a) $\vec{k}_h = -\vec{k}_e; \vec{v}_h = -\vec{v}_e; m_h = -m_e$ (b) $\vec{k}_h = \vec{k}_e; \vec{v}_h = \vec{v}_e; m_h = m_e$

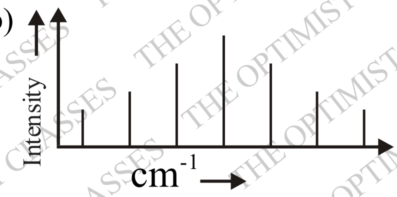
(b) $\vec{k}_h = \vec{k}_e; \vec{v}_h = -\vec{v}_e; m_h = -m_e$ (d) $\vec{k}_h = -\vec{k}_e; \vec{v}_h = \vec{v}_e; m_h = -m_e$

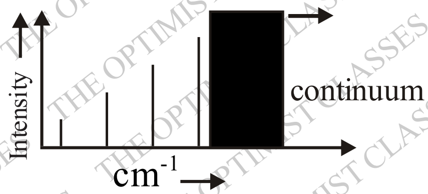
Q30. In a diatomic molecule, the internuclear separation of the ground and first excited electronic state the same as shown in the figure.

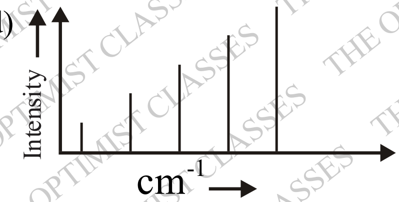


If the molecule is initially in the lowest vibrational state of the ground state, then the absorption spetrum will appear as

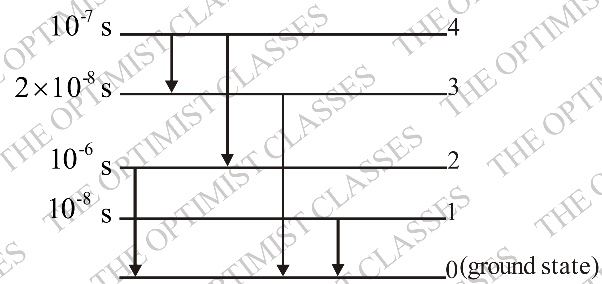
(a)


(b)


(c)


(d)


Q31. Five energy level of a system including the ground sate are shown below. Their lifetimes and the allowed electric dipole transitions are also marked.



Which one of the following transitions is the most suitable for a continuous wave (*CW*) laser?

(a) $1\to0$ (b) $2\to0$ (c) $4\to2$ (d) $4\to3$

Q32. Assuming the mean life time of a muon (in its rest frame) to be $2\times10^{-6}s$, its life time in the laboratory frame, when it is moving with a velocity $0.95c$ is

(a) $6.4\times10^{-6}s$ (b) $0.62\times10^{-6}s$ (c) $2.16\times10^{-6}s$ (d) $0.19\times10^{-6}s$

Q33. Cesium has a nuclear spin of 7/2. The hyperfine spectrum of the *D* lines of the Cesium atom will consist of

(a) 10 lines (b) 4 lines (c) 6 lines (d) 14 lines

Q34. The probability that an energy level ε at a temperature *T* is unoccupied by a fermion of chemical potential μ is given by

(a) $\dfrac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$ (b) $\dfrac{1}{e^{(\varepsilon-\mu)/k_BT}-1}$ (c) $\dfrac{1}{e^{(\mu-\varepsilon)/k_BT}+1}$ (d) $\dfrac{1}{e^{(\mu-\varepsilon)/k_BT}-1}$

Q35. Consider the follwing expression for the mass of a nucleus with *Z* protons and *A* nucleons.

$M(A,Z)=\dfrac{1}{c^2}\left[f(A)+yZ+zZ^2\right]$. Here, *f*(*A*) is a function of *A*.

$y=-4a_A$

$z=a_cA^{-1/3}+4a_AA^{-1}$

a_A and a_c are constants of suitable dimensions. For a fixed *A*, the expression of *Z* for the most stable nucleus is

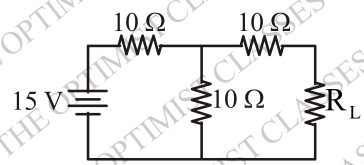
(a) $Z=\dfrac{A/2}{\left[1+\left(\dfrac{a_c}{a_A}\right)A^{2/3}\right]}$ (b) $Z=\dfrac{A/2}{\left[1+\left(\dfrac{a_c}{4a_A}\right)A^{2/3}\right]}$

(c) $Z = \frac{A}{\left[1+\left(\frac{a_c}{4a_A}\right)A^{2/3}\right]}$ (d) $Z = \frac{A}{\left[1+A^{2/3}\right]}$

Q36. The de Broglie wavelength of particles of mass m with average momentum p at a temperature T in three dimensioons is given by

(a) $\lambda = \frac{h}{\sqrt{2mk_BT}}$ (b) $\lambda = \frac{h}{\sqrt{3mk_BT}}$ (c) $\lambda = \frac{h}{\sqrt{2k_BT}}$ (d) $\lambda = \frac{h}{\sqrt{3m}}$

Q37. Assuming an ideal voltage source, Thevenin's resistance and Thevenin's voltage respectively for the below circuit are



(a) 15Ω and $7.5\,V$ (b) 20Ω and $5V$ (c) 10Ω and $10\,V$ (d) 30Ω and $15\,V$

Q38. Let $|n\rangle$ and $|p\rangle$ denote the isospin state with $I=\frac{1}{2}, I_3=\frac{1}{2}$ and $I=\frac{1}{2}, I_3=-\frac{1}{2}$ of a nuclear respectively. Which one of the following two nuclear state has $I=0, I_3=0$?

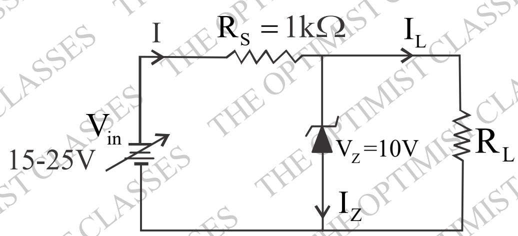
(a) $\frac{1}{\sqrt{2}}\left(|nn\rangle-|pp\rangle\right)$ (b) $\frac{1}{\sqrt{2}}\left(|nn\rangle+|pp\rangle\right)$ (c) $\frac{1}{\sqrt{2}}\left(|np\rangle-|pn\rangle\right)$ (d) $\frac{1}{\sqrt{2}}\left(|np\rangle+|pn\rangle\right)$

Q39. An amplifier of gain 1000 is made into a feedback amplifier by feeding 9.9% of its output voltage in series with the input opposing. If $f_L = 20\,Hz$ and $f_H = 200\,kHz$ for the amplifier without feedback, then due to the feed-back.

(a) The gain decrease by 10 times (b) The outputresistance increases by 10 times

(c) The f_H increases by 100 times (d) The input resistance decreases by 100 times.

Q40. Pick the correct statement based on the below circuit.



(a) The maximum Zener current, $I_{Z(\max)}$, when $R_L = 10k\Omega$ is $15mA$

(b) The minimum Zener current, $I_{Z(\min)}$, when $R_L = 10k\Omega$ is $5mA$

(c) With $V_{in} = 20V, I_L = I_Z$, when $R_L = 2k\Omega$

(d) The power dissipated across the Zener when $R_L = 10k\Omega$ and $V_{in} = 20V$ is $100\,mW$

Q41. The disintegration energy is defined to be the difference in the rest energy between the initial and final states. Consider the following process:

$$^{240}_{94}Pu \rightarrow ^{236}_{92}U + ^{4}_{2}He$$

The emitted α particle has a kinetic energy 5.17 MeV. The value of the disintegration energy is

(a) 5.26 MeV (b) 5.17 MeV (c) 5.08 MeV (d) 2.59 MeV

Q42. A classical particle is moving in an external potential field $V(x, y, z)$ which is invariant under the following infinitesimal transformations

$x \to x' = x + \delta x$,

$y \to y' = y + \delta y$,

$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x' \\ y' \end{pmatrix} = R_z \begin{pmatrix} x \\ y \end{pmatrix}$

where R_z is the matrix corresponding to rotation about the z-axis. The conserved quantities are (the symbols have their usual meaning)

(a) p_x, p_z, L_z (b) p_x, p_y, L_z, E (c) p_y, L_z, E (d) p_y, p_z, L_x, E

Q43. The spin function of a free particle, in the basis in which S_z is diagonal, can be written $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively. In the given basis, the normalized eigenfunction of S_y with eigenvlaue $-\frac{\hbar}{2}$

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$

Q44. $\hat{A}$ and $\hat{B}$ represents two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A}\hat{B}$ to be Hermitian, it is sufficient that

(a) $\hat{B}$ is Hermitian (b) $\hat{B}$ is anti-Hermitian

(c) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute (d) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ is anti-commute.

Q45. Consider the set of vectors in the three-dimensional real vectors space R^3, $S = \{(1,1,1), (1,-1,1), (1,1,-1)\}$. Which one of the following statements is true?

(a) S is not a linearly independent set (b) S is a basis for R^3

(c) The vectors in S are orthogonal (d) An orthogonal set of vectors cannot be generated from S

Q46. For a Fermi gas of N particles in three dimensional at $T = 0\,K$, the Fermi energy, E_F is proportional to

(a) $N^{2/3}$ (b) $N^{3/2}$ (c) N^3 (d) N^2

Q47. The Lagrangian of a diatomic molecule is given by $L = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{k}{2}x_1 x_2$, where m is the mass of each of the atoms and x_1 and x_2 are the displacements of atoms measured from the equilibrium position and $k > 0$. The normal frequencies are

(a) $\pm\left(\frac{k}{m}\right)^{1/2}$ (b) $\pm\left(\frac{k}{m}\right)^{1/4}$ (c) $\pm\left(\frac{k}{2m}\right)^{1/4}$ (d) $\pm\left(\frac{k}{2m}\right)^{1/2}$

Q48. A particle is in the normalized state $|\Psi\rangle$ which is a superposition of the energy eigenstates $|E_0 = 10eV\rangle$ and $|E_1 = 30eV\rangle$. The average value of energy of the particle in the state $|\Psi\rangle$ is $20eV$. The state $|\psi\rangle$ is given by

(a) $\frac{1}{2}|E_0 = 10eV\rangle + \frac{\sqrt{3}}{4}|E_1 = 30eV\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0 = 10eV\rangle + \sqrt{\frac{2}{3}}|E_1 = 30eV\rangle$

(c) $\frac{1}{2}|E_0 = 10eV\rangle - \frac{\sqrt{3}}{4}|E_1 = 30eV\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0 = 10eV\rangle - \frac{1}{\sqrt{2}}|E_1 = 30eV\rangle$

Q49. The Lagrangian of a particle of mass m moving in one dimension is $L = \exp(\alpha t)\left[\frac{m\dot{x}^2}{2} - \frac{kx^2}{2}\right]$, where α and k are positive constant. The equation of motion of the particle is

(a) $\ddot{x} + \alpha\dot{x} = 0$ (b) $\ddot{x} + \frac{k}{m}x = 0$ (c) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$ (d) $\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$

Q50. Two monochromatic waves having frequencies ω and $\omega + \Delta\omega(\Delta\omega << \omega)$ and corresponding wavelengths λ and $\lambda - \Delta\lambda(\Delta\lambda << \lambda)$ of same polarization, travelling along x-axis are superimposed on each other. The phase velocity and group velocity of the resultant wave are respesctively given by

(a) $\frac{\omega\lambda}{2\pi}, \frac{\Delta\omega\lambda^2}{2\pi\Delta\lambda}$ (b) $\omega\lambda \frac{\Delta\omega\lambda^2}{\Delta\lambda}$ (c) $\frac{\omega\Delta\lambda}{2\pi}, \frac{\Delta\omega\Delta\lambda}{2\pi}$ (d) $\omega\Delta\lambda, \omega\Delta\lambda$

Common data for Questions 51 and 52.

Consider a two level qunatum system with energies $\in_1 = 0$ and $\in_2 = \in$

Q51. The Helmholtz free energy of the system is given by

(a) $-k_B T \ \text{In}\left(1 + e^{-\varepsilon/k_B T}\right)$ (b) $k_B T \ \text{In}\left(1 + e^{-\varepsilon/k_B T}\right)$

(c) $\frac{3}{2}k_B T$ (d) $\varepsilon - k_B T$

Q52. The specific heat of the system is given

(a) $\frac{\varepsilon}{k_B T}\frac{e^{-\varepsilon/k_B T}}{\left(1 + e^{-\varepsilon/k_B T}\right)^2}$ (b) $\frac{\varepsilon^2}{k_B T^2}\frac{e^{-\varepsilon/k_B T}}{\left(1 + e^{-\varepsilon/k_B T}\right)}$ (c) $-\frac{\varepsilon e^{-\varepsilon/k_B T}}{\left(1 + e^{-\varepsilon/k_B T}\right)^2}$ (d) $\frac{\varepsilon^2}{k_B T^2}\frac{e^{-\varepsilon/k_B T}}{\left(1 + e^{-\varepsilon/k_B T}\right)^2}$

Common data for Questions 53 and 54 :

A free particle of mass m moves along the x-direction. At $t = 0$, the normalized wave function of the particle is given by $\psi(x,0)\frac{1}{\left(2\pi\alpha^2\right)^{1/4}}\exp\left[-\frac{x^2}{4\alpha^2} + ix\right]$, Where α is a real constant

Q53. The expectation value of the momentum, in this state is

(a) $\hbar\alpha$ (b) $\hbar\sqrt{\alpha}$ (c) α (d) $\frac{\hbar}{\sqrt{\alpha}}$

Q54. The expectation value of the particle energy is

(a) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{3/2}}$ (b) $\frac{\hbar^2}{2m}\alpha^2$ (c) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{3/2}}$ (d) $\frac{\hbar^2}{8m\alpha^{3/2}}$

Common Data for Questions 55 and 56 :

Consider the Zeeman splitting of a single electron system for the $3d \rightarrow 3p$ electric dipole transition.

Q55. The Zeeman spectrum is:

(a) Randomly polarized (b) Only π polarized

(c) Only σ polarized (d) Both π and σ polarized

Q56. The fine structure line having the longest wavelength will split into

(a) 17 components (b) 10 components (c) 8 components (d) 4 components

Statement for Linked Answer Questions 57 and 58 :

The primitive translation vectors of the *fcc* reciprocal cubic (*fcc*) lattice are

$$\vec{a}_1 = \frac{a}{2}\left(\hat{j}+\hat{k}\right), \vec{a}_2 = \frac{a}{2}\left(\hat{i}+\hat{k}\right), \vec{a}_3 = \frac{a}{2}\left(\hat{i}+\hat{j}\right)$$

Q57. The primitive translation vectors of the *fcc* reciprocal lattice are

(a) $\vec{b}_1 = \frac{2\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right); \vec{b}_2 = \frac{2\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right); \vec{b}_3 = \frac{2\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(b) $\vec{b}_1 = \frac{\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right); \vec{b}_2 = \frac{\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right); \vec{b}_3 = \frac{\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(c) $\vec{b}_1 = \frac{\pi}{2a}\left(-\hat{i}+\hat{j}+\hat{k}\right); \vec{b}_2 = \frac{\pi}{2a}\left(\hat{i}-\hat{j}+\hat{k}\right); \vec{b}_3 = \frac{\pi}{2a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(d) $\vec{b}_1 = \frac{3\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right); \vec{b}_2 = \frac{3\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right); \vec{b}_3 = \frac{3\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$

Q58. The volume of primitive cell of the *fcc* reciprocal lattice is

(a) $4\left(\frac{2\pi}{a}\right)^3$ (b) $4\left(\frac{\pi}{a}\right)^3$ (c) $4\left(\frac{\pi}{2a}\right)^3$ (d) $4\left(\frac{3\pi}{a}\right)^3$

Statement for Linked Answer Questions.59 and 60 :

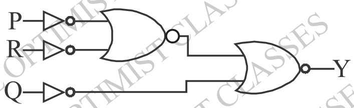
The Karnaugh map of a logic circuit is shown below :

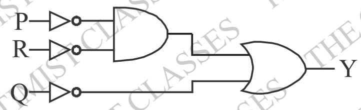
	$\bar{R}$	R
$\bar{P}\bar{Q}$	1	1
$\bar{P}Q$	1	
PQ		
$P\bar{Q}$	1	1

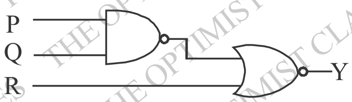
Q59. The minimized logic expression for the above map is :

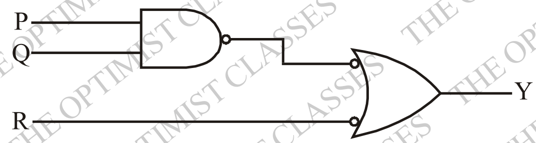
(a) $Y = \overline{P}\overline{R} + \overline{Q}$ (b) $Y = \overline{Q}.PR$ (c) $Y = \overline{Q} + PR$ (d) $Y = Q.\overline{PR}$

Q60. The corresponding logic implementation using gates is given as :

(a) 

(b) 

(c) 

(d) 

ANSWER KEY

1.	**(a)**	**2.**	**(c)**	**3.**	**(d)**	**4.**	**(d)**	**5.**	**(b)**
6.	**(d)**	**7.**	**(c)**	**8.**	**(b)**	**9.**	**(c)**	**10.**	**(b)**
11.	**(d)**	**12.**	**(a)**	**13.**	**(a)**	**14.**	**(a)**	**15.**	**(d)**
16.	**(b)**	**17.**	**(a)**	**18.**	**(b)**	**19.**	**(d)**	**20.**	**(c)**
21.	**(b)**	**22.**	**(b)**	**23.**	**(c)**	**24.**	**(c)**	**25.**	**(c)**
26.	**(a)**	**27.**	**(c)**	**28.**	**(b)**	**29.**	**(d)**	**30.**	**(a)**
31.	**(b)**	**32.**	**(a)**	**33.**	**(a)**	**34.**	**(c)**	**35.**	**(c)**
36.	**(b)**	**37.**	**(a)**	**38.**	**(c)**	**39.**	**(c)**	**40.**	**(c)**
41.	**(a)**	**42.**	**(b)**	**43.**	**(d)**	**44.**	**(c)**	**45.**	**(b)**
46.	**(a)**	**47.**	**(d)**	**48.**	**(d)**	**49.**	**(d)**	**50.**	**(a)**
51.	**(a)**	**52.**	**(d)**	**53.**	**(*)**	**54.**	**(*)**	**55.**	**(d)**
56.	**(b)**	**57.**	**(a)**	**58.**	**(a)**	**59.**	**(a)**	**60.**	**(b)**