THE OPTIMIST CLASSES
IIT-JAM TOPPERS

MANOJ KUMAR SINGH

ABHAY

PAWAN

SATYAM

SOUMIL GIRISH SAHU

BHOOMIJA

AKSHIT AGGARWAL

SHIKHAR CHAMOLI

RAVI SINGH ADHIKARI

GAURAV JHA

SWAPNIL JOSHI

LOKESH BHATT

GOPESH VISHVAKARMA

VAIBHAV

SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022

ANNU
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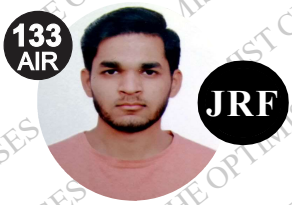

ALANKAR
UP15000162

SAHIL RANA
HR09000108

JAYESTHI
RJ11000161

DASRATH
RJ06000682

VIVEK
UK01000439

UZAIR AHMED
UP02000246

SURYA PRATAP SINGH
RJ06000232

HIMANSHU
UP10000095

CHANDAN
RJ09000159

SAIKHOM JOHNSON
MN01000196

AJAY SAINI
RJ06001744

VIKAS YADAV
RJ06001102

JYOTSNA KOHLI
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SHYAM SUNDAR
RJ06000615

GATE PAPER 2014

SECTION - A

Q1. A student is required to demonstrate a high level of comprehension of the subject, expecially in the social sciences.

The word closest in meaning to comprehension is

(a) understanding (b) meaning (c) concentration (d) stability

Q2. Choose the most appropriate word from the options given below to complete the following sentence.

One of his biggest_____was his ability to forgive.

(a) vice (b) virtues (c) choices (d) strength

Q3. Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can inferred from the above sentences?

(a) Rajan has decided to work only in a group

(b) Rajan and Sajan were formed into a group against their wishes

(c) Sajan had decided to given in to Rajan's request to work with him

(d) Rajan had beliveved that Sajan and he would be working together

Q4. If $y = 5x^2 + 3$, then the tangent $x = 0$, $y = 3$

(a) passes through $x = 0$, $y = 0$ (b) has a slope of + 1

(c) is parallel to the x-axis (d) has a slope of – 1

Q5. A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

Q.6 – Q.10 Carry Two marks each

Q6. Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

(a) ALRVX (b) EPVZB (c) ITZDF (d) OYEIK

Q7. Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives

three floors above Dilip. Which of the following floor-person combinations is correct?

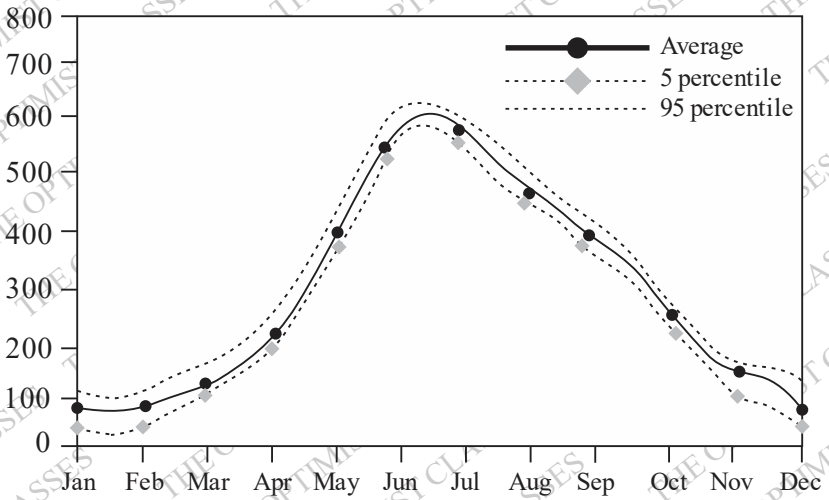
	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

Q8. The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3 : 4 : 5 : 6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

Q9. One percent of the people of country *X* are taller than 6 ft. Two percent of the people of country *Y* are taller than 6 ft. There are thrice as many people in country *X* as in country *Y*. Taking both countries together, what is the percentage of people taller then 6 ft?

(a) 3.0 (b) 2.5 (c) 1.5 (d) 1.25

Q10. The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (*k* percentile is the value such that *k* percent of the data fall below that value)



(i) On average, it rains more in July than in December

(ii) Every year, the amount of rainfall in August is more than that is January

(iii) July rainfall can be estimated with better confidence than February rainfall

(iv) In August, there is at least 500*mm* of rainfall

(a) (i) and (ii) (b) (i) and (iii) (c) (ii) and (iii) (d) (iii) and (iv)

SECTION - B

Q.1- Q.25 : Carry ONE mark each.

Q1. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1) is

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

Q2. Which one of the following quantities is invariant under Lorentz transformation?
(a) Charge density (b) Charge (c) Current (d) Electric field

Q3. The number of normal Zeeman splitting components of ${}^1P \rightarrow {}^1D$ transition is
(a) 3 (b) 4 (c) 8 (d) 9

Q4. If the half-life of an elementary particle moving with speed $0.9c$ in the laboratory frame is $5\times10^{-8}\,s$, then the proper half-life is _____$\times10^{-8}\,s$. $(c = 3\times10^{8}\,m/s)$

Q5. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is
(a) $0°$ (b) $45°$ (c) $90°$ (d) $120°$

Q6. Two masses m and $3m$ are attached to the two ends of a massless spring with force constant K. If $m = 100g$ and $K = 0.3\ N/m$, then the natural angular frequency of oscillation is _____Hz.

Q7. The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E} = \hat{x}10\cos\left(6\pi\times10^{7}t - 0.4\pi z\right)V/m$$

The phase velocity of the wave is _____$\times10^{8}\,m/s$.

Q8. The matrix $A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix}$ is

(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

Q9. The recoil momemtum of an atom is p_A when it emits an infrared photon of wavelength 1500nm, and it is P_B when it emits a photon of visible Wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

Q10. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to
(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

Q11. Which one of the following high energy processes is allowed by conservation laws ?
(a) $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$ (b) $\pi^- + p \rightarrow \pi^0 + n$
(c) $n \rightarrow p + e^- + \nu_e$ (d) $\mu^+ \rightarrow e^- + \gamma$

Q12. The Length element ds of an arc is given by, $(ds)^2 = 2\left(dx^1\right)^2 + \left(dx^2\right)^2 + \sqrt{3}dx^1dx^2$. The metric tensor g_{ij} is

(a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

Q13. The ground state and the first excited state wave function of a one dimensional infinite potential well are ψ_1 and ψ_2 respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)-\psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)+\psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)+\psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)-\psi_1(x_2)\psi_2(x_1)\right]$

Q14. If the vector potential $\vec{A}=\alpha x\hat{x}+2y\hat{y}-3z\hat{z}$ Satisfies the Coulomb gauge, the value of the constant α is _____

Q15. At a given temperature *T,* the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillator is _____ k_BT.

Q16. Which one of the following is a fermion ?

(a) α particle (b) ${}_4Be^7$ nucleus (c) hydrogen atom (d) deuteron

Q17. Which one of the following three-quark states (qqq), denoted by *X*, CANNOT be a possible baryon ? The corresponding electric charge is indicated in the superscript

(a) X^{++} (b) X^{+} (c) X^{-} (d) X^{--}

Q18. The Hamilton's canonical equations of motion in terms of poisson Brackets are

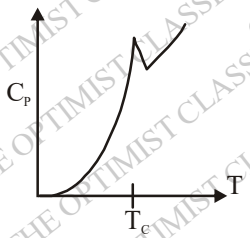
(a) $\dot{q}=\{q,H\};\dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\};\dot{p}=\{H,q\}$

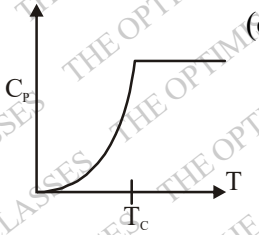
(c) $\dot{q}=\{H,p\};\dot{p}=\{H,q\}$ (d) $\dot{q}=\{p,H\};\dot{p}=\{q,H\}$

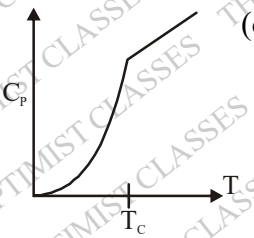
Q19. The Miler indices of a plane passing through the three points having coordinates (0,0,1), (1,0,0), $\left(\frac{1}{2},\frac{1}{2},\frac{1}{4}\right)$ are

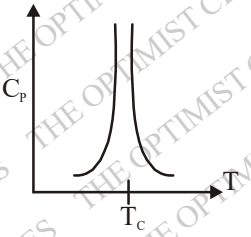
(a) (2 1 2) (b) (1 1 1) (c) (1 2 1) (d) (2 1 1)

Q20. The plot of specific heat versus temperature across the superconducting transition temperature (T_C) is most appropriately represented by

(a) 

(b) 

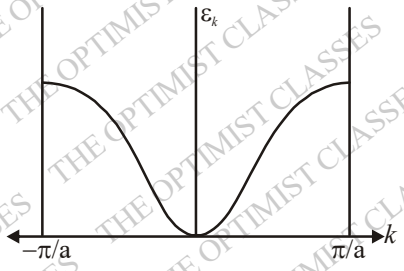
(c) 

(d) 

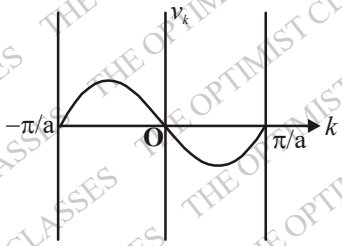
Q21. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular mometum, then $\vec{L}.\vec{S}$ does not commute with

(a) S_z (b) L^2 (c) S^2 (d) $\left(\vec{L}+\vec{S}\right)^2$

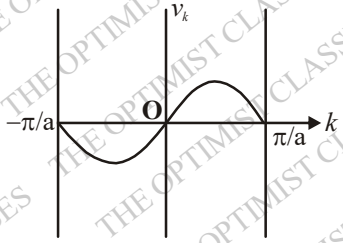
Q22. The energy, ε_k for band electrons as a function of the wave vector, *k* in the first Brillouin zone $\left(-\frac{\pi}{a}\le k\le\frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as ('*a*' is lattice constant)



(a)



(b)



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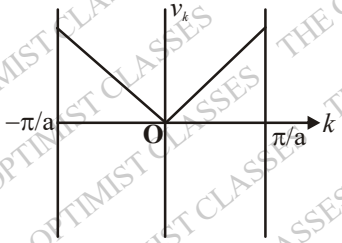
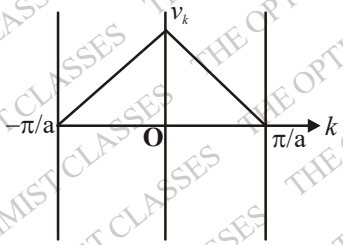
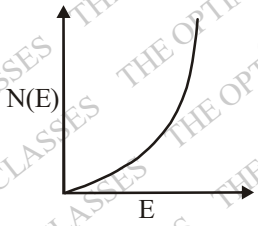


(d)

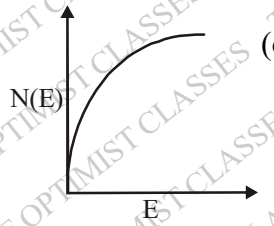


Q23. For a free electron gas in two dimensions, the variation of the states, $N(E)$ as a function of energy E, is the best represented by

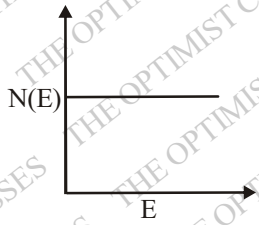
(a)



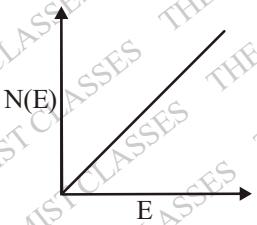
(b)



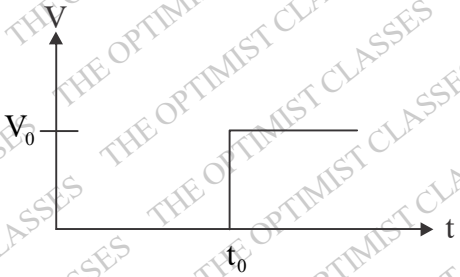
(c)



(d)

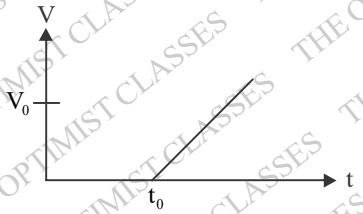


Q24. The input given to be an ideal OP-AMP integrator circuit is

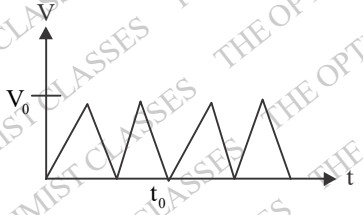


The correct output of the integrator circuit is (in magnitude form)

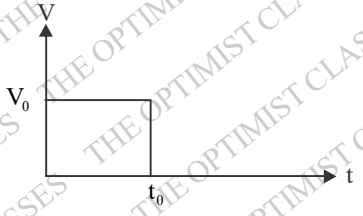
(a)



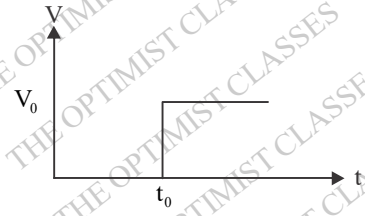
(b)



(c)



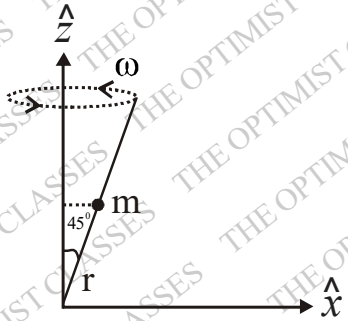
(d)



Q25. The minimum number of flip-flops required to construct a mod-75 counter is _____

Q.26 - Q55 : Carry TWO maks each.

Q26. A bead of mass '*m*' can slide without friction along a massless rod kept at 45^0 with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω . At any instant, *r* is the distance of the bead from the origin. The momentum conjugate to *'r'* is



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

Q27. An electron in the ground state of the hydrogen atom has the wavefunction $\psi\left(\vec{r}\right)=\frac{1}{\sqrt{\pi a_0^3}}e^{-(r/a_0)}$ where a_0 is constant. The expectation value of the operator $\hat{Q}=z^2-r^2$, where $z=r\cos\theta$ is

(Hint: $\int_0^\infty e^{-ar}r^n dr=\frac{\Gamma(n)}{a^{n+1}}=\frac{(n-1)!}{a^{n+1}}$)

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$ (c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

Q28. For Nickel, the number density is 8×10^{23} atoms/cm^3 and electronic configuration is $1s^2 2s^2 2p^6 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic states is _____ $\times10^9 A/m$.

Q29. A particle of mass '*m*' is in a potential given by $V(x)=-\frac{a}{r}+\frac{ar_0^2}{3r^3}$ where '*a*' and 'r_0' are positive constants.

When distrubed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillaion is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

Q30. The donor concentration in a sample of *n*-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300*K*, assuming the sample to be non degenerate is _____ meV. $\left(k_BT = 25meV \text{ at } 300\,K\right)$

Q31. A particle of mass '*m*' is subjected to a potential, $V(x,y) = \frac{1}{2}m\omega^2\left(x^2+y^2\right), -\infty \le x \le \infty, -\infty \le y \le \infty$ The state with energy is *g*-fold degenerate. The value of *g* is _____

Q32. A hydrogen atom is in the state $\psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$ where n, ℓ, m in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is _____ $\hbar^2$.

Q33. A planet of mass *m* moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$ where *k* is a positive constant. The orbital angular momentum of the palnet is

(a) $2r_0km$ (b) $\sqrt{2r_0km}$ (c) r_0km (d) $\sqrt{r_0km}$

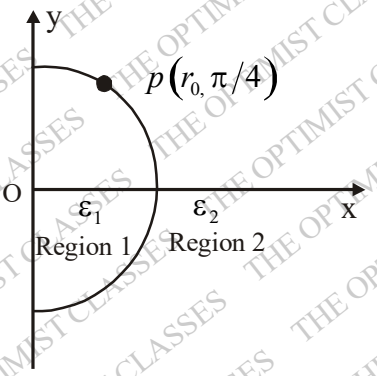
Q34. The moment of inertia of a rigid diatomic molecule *A* is 6 times that of another rigid diatomic moecule *B*. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$.

Q35. The value of the integral $\oint_C \frac{z^2}{e^z+1}dz$ where *C* is the circle $|z| = 4$, is

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

Q36. A ray of light inside region 1 in the *xy*-plane is incident at the semicircle boundary that carries no free charges. The electric field at the point $P\left(r_0, \pi/4\right)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in region 2 has the electric field $\vec{E}_2$ parallel to *x*-axis. If ε_1 and ε_2 are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is _____.



Q37. The solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$ subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$ is

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

Q38. Given that the linear transformation of a generalized coordinate 'q' and the corresponding momentum p,

$$Q = q + 4ap$$
$$P = q + 2p$$

is canonical, the value of the constant 'a' is _____

Q39. The value of the magnetic field required to maintain non-relativistic protons of energy $1MeV$ in a circular orbit of radius $100mm$ is _____ Tesla. ($Given: m_p = 1.67\times10^{-27}\,kg, e = 1.67\times10^{-19}\,C$)

Q40. For a system of two bosons, each of which can occupy any of two energy levels 0 and ε, the mean energy of the system at a temperature T with $\beta = \frac{1}{k_B T}$ is given by

(a) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (b) $\frac{1 + \varepsilon e^{-\beta\varepsilon}}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (c) $\frac{2\varepsilon e^{-\beta\varepsilon} + \varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon e^{-\beta\varepsilon} + 2\varepsilon e^{-2\beta\varepsilon}}{2 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

Q41. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and I_0, respectively. The intensities of the individual waves are

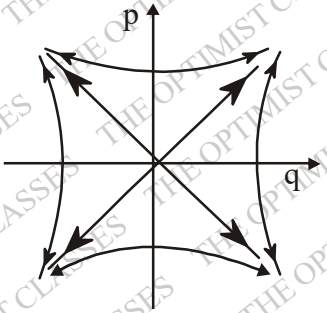
(a) $3I_0$ and I_0 (b) $4I_0$ and I_0 (c) $5I_0$ and $4I_0$ (d) $9I_0$ and I_0

Q42. ψ_1 and ψ_2 are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that $\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$ where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represents the spin-up and spin-down states, respectively. When the system is in the state ψ_2, its probability to be in the spin-up state is _____

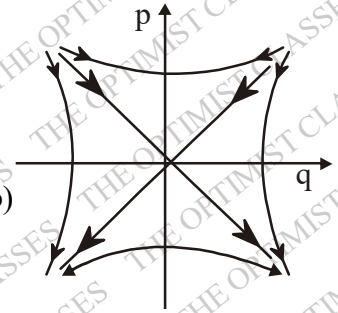
Q43. Neutrons moving with speed $10^3\ m/s$ are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is $30°$, the interplanar spacing of the crstal is _____ $\mathring{A}$.

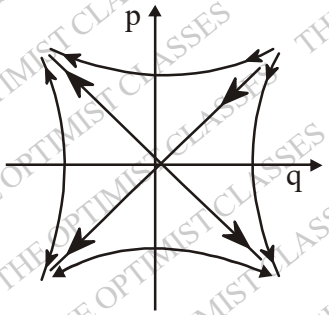
Q44. The Hamiltonian of a particle of mass 'm' is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which of the follwing figures describes the motion of the particle in phase space?

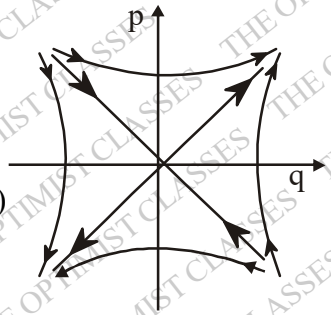
(a)



(b)



(c) 

(d) 

Q45. The intensity of a laser in free space is 150 $m\,W/m^2$. The corresponding amplitude of the electric field of the laser is _____ V/m. $\left(\varepsilon_0 = 8.854\times10^{-12} C^2/N.m^2\right)$

Q46. The emission wavelength for the transition ${}^1D_2 \rightarrow {}^1F_3$ is 3122 Å. The ratio of populations of the final to the initial states at a temperature 5000 K is $\left(h = 6.626\times10^{-34} Js, c = 3\times10^8 m/s, k_B = 1.380\times10^{-23} J/K\right)$

(a) 2.03×10^{-5} (b) 4.02×10^{-5} (c) 7.02×10^{-5} (d) 9.83×10^{-5}

Q47. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) k_B In 2 (b) $2k_B$ In 2 (c) $2k_B$ In 4 (d) $3k_B$ In 4

Q48. A particle is confined in a one-dimensional potential box with the potential

$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

If the particle is subjected to a perturbation, within the box, $W = \beta x$ where β is a small constant, the first order correction to the ground state energy is

(a) 0 (b) $\frac{a\beta}{4}$ (c) $\frac{a\beta}{2}$ (d) $a\beta$

Q49. Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion (π) pairs at rest is _____ MeV.

(Given : $m_\mu = 105\ MeV/c^2, m_\pi = 140\ MeV/c^2$)

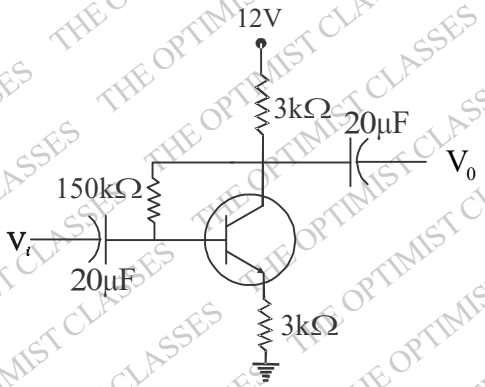
Q50. A one dimensional harmonic oscillator is in the superposition of number states, $|n\rangle$, given by

$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$. The average energy of the oscillator in the given state is _____ $\hbar\omega$.

Q51. A nucleus X undergoes a first forbidden β-decay to a nucleus Y. If the angular momentum (I) and parity (P), denoted by I^P as $\frac{7^-}{2}$ for X, which of the following is a possible I^P value for Y?

(a) $\frac{1^+}{2}$ (b) $\frac{1^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{3^-}{2}$

Q52. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_C is _____ mA.



Q53. In order to measure a maximum of $1V$ with a resolution of $1mV$ using a n-vit A/D converter, working under. The principle of ladder network, the minimum value of n is _____

Q54. If L_+ and L_- are the angular momentum ladder operators, then the expectation value of $(L_+L_- + L_-L_+)$ in the state $|l=1, m=1\rangle$ of an atom is _____ $\hbar^2$.

Q55. A low pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively, are

(a) 0.71 and 45° (b) 0.71 and –45° (c) 0.5 and –90° (d) 0.5 and 90°

ANSWER KEY

SECTION - A

1.	**(a)**	**2.**	**(b)**	**3.**	**(d)**	**4.**	**(c)**		
5.	**(1300 to 1300)**			**6.**	**(d)**	**7.**	**(b)**		
8.	**(180 to 180)**	**9.**	**(d)**	**10.**	**(b)**				

SECTION - B

1.	**(d)**	**2.**	**(b)**	**3.**	**(a)**	**4.**	**(2.1 to 2.3)**	**5.**	**(c)**
6.	**(1.99 to 2.01)**			**7.**	**(1.49 to 1.51)**			**8.**	**(d)**
9.	**(c)**	**10.**	**(d)**	**11.**	**(b)**	**12.**	**(b)**	**13.**	**(d)**
14.	**(0.99 to 1.01)**	**15.**	**(1.99 to 2.01)**	**16.**	**(b)**	**17.**	**(d)**	**18.**	**(a)**
19.	**(c)**	**20.**	**(a)**	**21.**	**(a)**	**22.**	**(b)**	**23.**	**(c)**
24.	**(a)**	**25.**	**(6.99 to 7.01)**	**26.**	**(a)**	**27.**	**(d)**		
28.	**(40 to 43)**	**29.**	**(a)**	**30.**	**(114 to 117)**	**31.**	**(3.99 to 4.01)**		
32.	**(1.99 to 2.01)**	**33.**	**(d)**	**34.**	**(b)**	**35.**	**(c)**		
36.	**(2.3 to 2.4)**	**37.**	**(d)**	**38.**	**(0.24 to 0.26)**	**39.**	**(1.41 to 1.47)**	**40.**	**(a)**
41.	**(b)**	**42.**	**(0.66 to 0.68)**	**43.**	**(3.91 to 4.15)**	**44.**	**(d)**		
45.	**(10.58 to 10.70)**			**46.**	**(c)**	**47.**	**(b)**	**48.**	**(c)**
49.	**(34.9 to 35.1)**	**50.**	**(3.2 to 3.3)**	**51.**	**(c)**	**52.**	**(1.4 to 1.7)**		
53.	**(9.99 to 10.01)**			**54.**	**(1.99 to 2.01)**	**55.**	**(b)**		