

THE OPTIMIST CLASSES IIT-JAM TOPPERS



MANOJ KUMAR SINGH



ABHAY



PAWAN



SATYAM



SOUMIL GIRISH SAHU



BHOOMIJA



AKSHIT AGGARWAL



SHIKHAR CHAMOLI



RAVI SINGH ADHIKARI



GAURAV JHA



SWAPNIL JOSHI



LOKESH BHATT



GOPESH VISHVAKARMA



VAIBHAV



SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022



ANNU
DL01000308



ALANKAR
UP15000162



SAHIL RANA
HR09000108



JAYESTHI
RJ11000161



DASRATH
RJ06000682



VIVEK
UK01000439



UZAIR AHMED
UP02000246



SURYA PRATAP SINGH
RJ06000232



HIMANSHU
UP10000095



CHANDAN
RJ09000159



SAIKHOM JOHNSON
MN01000196



AJAY SAINI
RJ06001744



VIKAS YADAV
RJ06001102



JYOTSNA KOHLI
UK02000262



SHYAM SUNDAR
RJ06000615

THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

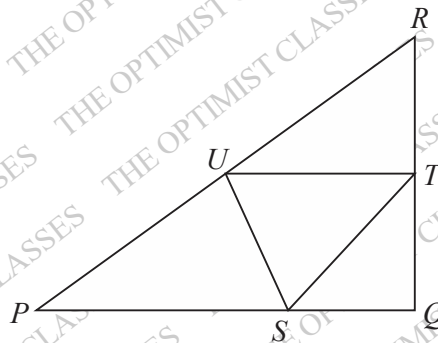
CONTACT: 9871044043

GATE PAPER 2015

SECTION - A

- Q1. Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence.
Apparent lifelessness _____ dormant life
(a) harbours (b) leads to (c) supports (d) affects
- Q2. Fill in the blank with the correct idiom/phrase
That boy from the town was a _____ in the sleepy village
(a) dog out of herd (b) sheep from the heap
(c) fish out of water (d) bird from the flock
- Q3. Choose the statement where underlined word is used correctly
(a) When the teacher eludes to different authors, he is being elusive
(b) When the thief keeps eluding the police, he is being elusive
(c) Matters that are difficult to understand, identify or remember are allusive
(d) Mirages can be allusive, but a better way to express them is illusory
- Q4. Tanya is older than Eric
Cliff is older than Tanya
Eric is older than Cliff
If the first two statements are true, then the third statement is
(a) True (b) False (c) Uncertain (d) Data insufficient
- Q5. Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?
(a) 20 (b) 10 (c) 8 (d) 5
- Q6. Select the appropriate option in place of underlined part of the sentence
Increased productivity necessary reflects greater efforts made by the employees
(a) Increase in productivity necessary
(b) Increases productivity is necessary
(c) Increase in productivity necessarily
(d) No improvement required
- Q7. Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.
Statements
I. No manager is a leader
II. All leaders are executives
Conclusions
I. No manager is an executive
II. No executive is a manager
(a) Only conclusion I follows (b) Only conclusion II follows
(c) Neither conclusion I nor II follows (d) Both conclusions I and II follow
- Q8. In the given figure angle Q is a right angle, $PS : QS = 3 : 1$, $RT : QT = 5 : 2$ and $PU : UR = 1 : 1$. If area of

triangle QTS is 20cm^2 , then the area of triangle PQR in cm^2 is _____



Right triangle PQR is to be constructed in the xy -plane so that the right angle is at P and line PR is parallel to the x -axis. The x and y coordinates of P, Q and R are to be integers that satisfy the inequalities: $-4 \leq x \leq 5$ and $6 \leq y \leq 16$. How many different triangles could be constructed with these properties?

- (a) 110 (b) 1,100 (c) 9,900 (d) 10,000

Q9. A coin is tossed thrice. Let X be the event that head occurs in each of the first two tosses. Let Y be the event that a tail occurs on the third toss. Let Z be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

- (a) X and Y are not independent (b) Y and Z are dependent
(c) Y and Z are independent (d) X and Z are independent

Q10. A satellite is moving in a circular orbit around the Earth. If T , V and E are its average kinetic average potential and total energies, respectively, then which one of the following options is correct?

- (a) $V = -2T$, $E = -T$ (b) $V = -T$, $E = 0$
(c) $V = T/2$, $E = T/2$ (d) $V = -3T/2$, $E = -T/2$

SECTION - B

Q.1 - Q.25 : Carry ONE mark each.

Q1. A satellite is moving in a circular orbit around the Earth. If T , V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

- (a) $V = -2T$; $E = -T$ (b) $V = -T$; $E = 0$ (c) $V = \frac{-T}{2}$; $E = \frac{T}{2}$ (d) $V = \frac{-3T}{2}$; $E = \frac{-T}{2}$

Q2. The lattice parameters a, b, c of an orthorhombic crystal are related by $a = 2b = 3c$. In units of a , the interplanar separation between the (110) planes is _____ (upto three decimal places).

Q3. Consider $w = f(z) = u(x, y) + iv(x, y)$ to be an analytic function in a domain D . Which one of the following options is NOT correct?

- (a) $u(x, y)$ satisfies Laplace equation in D
(b) $v(x, y)$ satisfies Laplace equation in D
(c) $\int_{z_1}^{z_2} f(z) dz$ is dependent on the choice of the contour between z_1 and z_2 in D
(d) $f(z)$ can be Taylor expanded in D

Q4. Let \bar{L} and \bar{p} be the angular and linear momentum operators respectively, for a particle. The commutator $[L_x, p_y]$ gives

- (a) $-i\hbar p_z$ (b) 0 (c) $i\hbar p_x$ (d) $i\hbar p_z$

Q5. The dispersion relation for photons in a one dimensional monatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by, $\omega(k) = \sqrt{\frac{2C}{M} [1 - \cos(ka)]}$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes ($\lambda \gg a$), the ratio of the phase velocity to the group velocity is _____.

Q6. For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because
 (a) the chemical potential of the photons is zero
 (b) photons obey Pauli exclusion principle
 (c) photons are spin - 1 particles
 (d) the entropy of the photons is very large

Q7. Four forces are given below in Cartesian and spherical polar coordinates.

(i) $\vec{F}_1 = K \exp\left(\frac{-r^2}{R^2}\right) \hat{r}$

(ii) $\vec{F}_2 = K (x^3 \hat{y} - y^3 \hat{z})$

(iii) $\vec{F}_3 = K (x^3 \hat{x} + y^3 \hat{y})$

(iv) $\vec{F}_4 = K \left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.

- (a) (iii) and (iv) are conservative but (i) and (ii) are not
 (b) (i) and (ii) are conservative but (iii) and (iv) are not
 (c) (ii) and (iii) are conservative but (i) and (iv) are not
 (d) (i) and (iii) are conservative but (ii) and (iv) are not

Q8. The value of $\int_0^3 t^2 \delta(3t-6) dt$ is _____ (upto one decimal place)

Q9. The mean kinetic energy of a nucleon in a nucleus of atomic weight A varies as A^n , where n is _____ (upto two decimal places)

Q10. In Bose-Einstein condensates, the particles

- (a) have strong interparticle attraction (b) condense in real space
 (c) have overlapping wavefunctions (d) have large and positive chemical potential

Q11. A beam of X-ray of intensity I_0 is incident normally on a metal sheet of thickness 2mm . The intensity of the transmitted beam is $0.025 I_0$. The linear absorption coefficient of the metal sheet (in m^{-1}) is _____ (upto one decimal place)

Q12. In a Hall effect experiment, the Hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

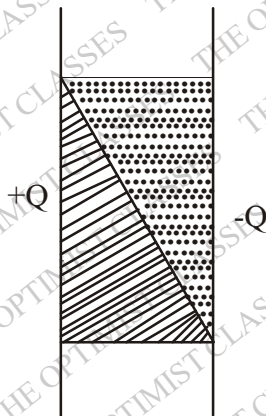
- (a) $n \approx p$ (b) $n > p$ (c) $\mu_e > \mu_h$ (d) $m_e^* > m_h^*$

Q13. The Pauli matrices for three spin $-\frac{1}{2}$ particles are $\vec{\sigma}_1, \vec{\sigma}_2$ and $\vec{\sigma}_3$ respectively. The dimension of the Hilbert space required to define an operator is $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is _____

Q14. The decay $\mu^+ \rightarrow e^+ + \gamma$ is forbidden, because it violates

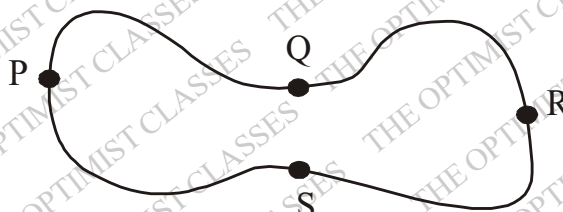
- (a) momentum and lepton number conservations
 (b) baryon and lepton number conservations
 (c) angular momentum conservation
 (d) lepton number conservation

Q15. The space between two plates of a capacitor carrying charges $+Q$ and $-Q$ is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct ?



- (a) \vec{E} and \vec{D} are continuous
 (b) \vec{E} is continuous and \vec{D} is discontinuous
 (c) \vec{D} is continuous and \vec{E} is discontinuous
 (d) \vec{E} and \vec{D} are discontinuous

Q16. Given that magnetic flux through the closed loop PQRSP is ϕ . If $\int_P^R \vec{A} \cdot d\vec{l} = \phi_1$ along PQR, the value of $\int_P^R \vec{A} \cdot d\vec{l}$ along PSR is



- (a) $\phi - \phi_1$
 (b) $\phi_1 - \phi$
 (c) $-\phi_1$
 (d) ϕ_1

Q17. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is _____

Q18. Consider a complex function $f(z) = \frac{1}{z \left(z + \frac{1}{2} \right) \cos(z\pi)}$, Which one of the following statements is correct?

- (a) $f(z)$ has simple poles at $z=0$ and $z = -\frac{1}{2}$
 (b) $f(z)$ has a second order pole at $z = -\frac{1}{2}$
 (c) $f(z)$ has infinite number of second order poles

(d) $f(z)$ has all simple poles

Q19. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by,

$g(E) = CE^n$ where C is constant. The value of n is _____

Q20. In an inertial frame S , two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively.

The times at which these events take place in a frame S' moving with a velocity $0.6c\hat{y}$ with respect to S are given by

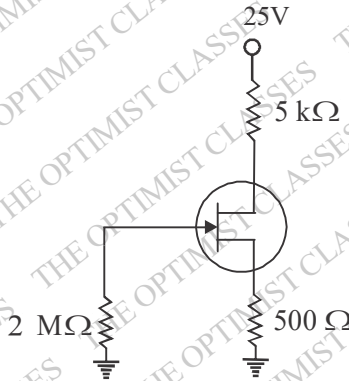
(a) $ct'_A = 0; ct'_B = -3/2$

(b) $ct'_A = 0; ct'_B = 0$

(c) $ct'_A = 0; ct'_B = 3/2$

(d) $ct'_A = 0; ct'_B = 1/2$

Q21. In the given circuit, the voltage across the source resistor is $1V$. The drain voltage (in V) is _____



Q22. If $f(x) = e^{-x^2}$ and $g(x) = |x|e^{-x^2}$

(a) f and g are differentiable everywhere

(b) f is differentiable everywhere but g is not

(c) g is differentiable everywhere but f is not

(d) g is discontinuous at $x = 0$

Q23. Consider a system of N non-interacting spin $-\frac{1}{2}$ particles, each having a magnetic moment μ , is in a magnetic

field $\vec{B} = B\hat{z}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

(a) $\Omega = \frac{N!}{2 \binom{N - \frac{E}{\mu B}}{2} \binom{N + \frac{E}{\mu B}}{2}}$

(b) $\Omega = \frac{\binom{N - \frac{E}{\mu B}}{2}}{\binom{N + \frac{E}{\mu B}}{2}}$

(c) $\Omega = \frac{1}{2} \binom{N - \frac{E}{\mu B}}{2} \binom{N + \frac{E}{\mu B}}{2}$

(d) $\Omega = \frac{N!}{\binom{N + \frac{E}{\mu B}}{2}}$

Q24. Which one of the following DOES NOT represent an exclusive OR operation for inputs A and B ?

(a) $(A + B)\overline{AB}$

(b) $A\overline{B} + B\overline{A}$

(c) $(A + B)(\overline{A} + \overline{B})$

(d) $(A + B)AB$

Q25. An operators for a spin $-1/2$ particle is given by $\hat{A} = \lambda \vec{\sigma} \cdot \vec{B}$, where $\vec{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{y})$, $\vec{\sigma}$ denotes Pauli matrix-

ces and λ is a constant. The eigenvalue of \hat{A} are

- (a) $\pm \frac{\lambda B}{\sqrt{2}}$ (b) $\pm \lambda B$ (c) $0, \lambda B$ (d) $0, -\lambda B$

Q.26 - Q.55 : Carry TWO marks each.

Q26. Match the phrases in **Group I** and **Group II** and identify the correct option.

Group I

Group II

- | | |
|---|------------------------------------|
| (P) Electronic spin resonance (ESR) | (i) radio frequency |
| (Q) Nuclear magnetic resonance (NMR) | (ii) visible range frequency |
| (R) Transition between vibrational states of a molecule | (iii) microwave frequency |
| (S) Electronic transition | (iv) far-infrared range |
| (a) (P-i), (Q-ii), (R-iii), (S-iv) | (b) (P-ii), (Q-i), (R-iv), (S-iii) |
| (c) (P-iii), (Q-iv), (R-i), (S-ii) | (d) (P-iii), (Q-i), (R-iv), (S-ii) |

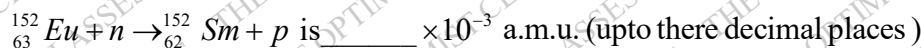
Q27. The entropy of a gas containing N particles enclosed in a volume V is given by $S = Nk_B \ln \left(\frac{aVE^{3/2}}{N^{5/2}} \right)$, where

E is the total energy, a is a constant and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by

(a) $\mu = -k_B T \left[\ln \left(\frac{aVE^{3/2}}{N^{5/2}} \right) - \frac{5}{2} \right]$ (b) $\mu = -k_B T \left[\ln \left(\frac{aVE^{3/2}}{N^{5/2}} \right) - \frac{3}{2} \right]$

(c) $\mu = -k_B T \left[\ln \left(\frac{aVE^{3/2}}{N^{3/2}} \right) - \frac{5}{2} \right]$ (d) $\mu = -k_B T \left[\ln \left(\frac{aVE^{3/2}}{N^{3/2}} \right) - \frac{3}{2} \right]$

Q28. The atomic masses of $^{152}_{63}\text{Eu}$, $^{152}_{62}\text{Sm}$, ^1_1H and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (a.m.u.), respectively. Using the above information, the Q -value of the reaction



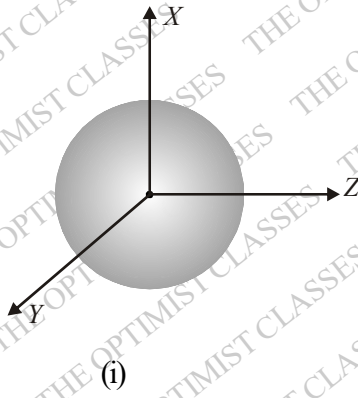
Q29. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z -axis. Their velocities are given by

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

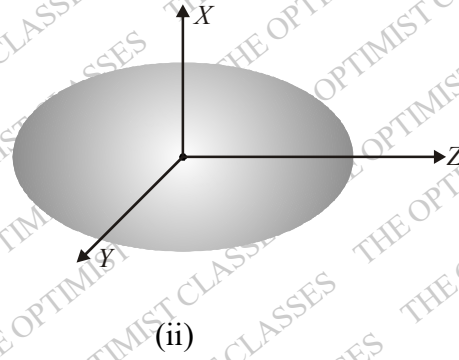
(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z}; \vec{v}_2 = (-0.8c)\hat{z}$

Q30. The band gap of an intrinsic semiconductor is $E_g = 0.72\text{eV}$ and $m_h^* = 6m_e^*$. At 300K , the Fermi level with respect to the edge of the valence band (in eV) is at $\underline{\hspace{2cm}}$ (upto three decimal places) $k_B = 1.38 \times 10^{-23} \text{JK}^{-1}$

Q31. A charge $-q$ is distributed uniformly over a sphere, with a positive charge q at its centre in (i). Also in (ii), a charge $-q$ is distributed uniformly over an ellipsoid with a positive charge q at its center. With respect to the origin of the coordinate system, which one of the following statements is correct?



(i)

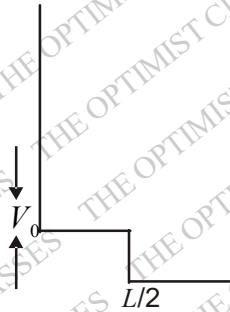


(ii)

- (a) The dipole moment is zero in both (i) and (ii)
 (b) The dipole moment is non-zero in (i) but zero in (ii)
 (c) The dipole moment is zero in (i) but non-zero in (ii)
 (d) The dipole moment is non-zero in both (i) and (ii)

Q32. The number of permitted transitions from ${}^2P_{3/2} \rightarrow {}^2S_{1/2}$ in the presence of a weak magnetic field is _____

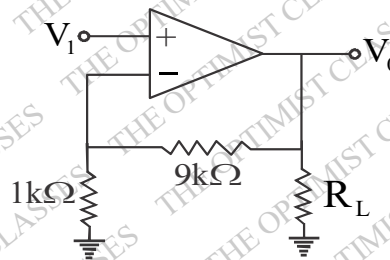
Q33. A particle is confined in a box of length L as shown below.



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

- (a) $E = \frac{\hbar^2 \pi^2}{2mL^2} + V_0$ (b) $E = \frac{\hbar^2 \pi^2}{2mL^2} - \frac{V_0}{2}$ (c) $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{4}$ (d) $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{2}$

Q34. In the given circuit, if the open loop gain $A = 10^5$, the feedback configuration and the closed loop gain A_f are



- (a) series-shunt, $A_f = 9$ (b) series-series, $A_f = 10$
 (c) series-shunt, $A_f = 10$ (d) shunt-shunt, $A_f = 10$

Q35. A plane wave $(\hat{x} + i\hat{y})E_0 \exp[i(kz - \omega t)]$ after passing through an optical element emerges as $(\hat{x} - i\hat{y})E_0 \exp[i(kz - \omega t)]$, where k and ω are the wavevector and the angular frequency, respectively. The

optical element is a

- (a) quarter wave plane
(c) polarizer

- (b) half wave plate
(d) Faraday rotator

Q36. A particle of mass 0.01kg falls freely in the earth's gravitational field with an initial velocity $v(0) = 10\text{ms}^{-1}$. If the air exerts a functional force of the form, $f = -kv$, then for $k = 0.05\text{Nm}^{-1}\text{s}$, the velocity (in Nm^{-1}) at time $t = 0.2\text{s}$ is _____ (upto two decimal places)

(use $g = 10\text{ms}^{-2}$ and $e = 2.72$)

Q37. The Lagrangian for a particle of mass m at a position \vec{r} moving with a velocity \vec{v} is given by $L = \frac{m}{2}\vec{v}^2 + C\vec{r} \cdot \vec{v} - V(r)$, where $V(r)$ is a potential and C is a constant. If \vec{p}_c is the canonical momentum, then its Hamiltonian is given by

(a) $\frac{1}{2m}(\vec{p}_c + C\vec{r})^2 + V(r)$

(b) $\frac{1}{2m}(\vec{p}_c - C\vec{r})^2 + V(r)$

(c) $\frac{p_c^2}{2m} + V(r)$

(d) $\frac{1}{2m}p_c^2 + C^2r^2 + V(r)$

Q38. A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance r from the axis of the solenoid is proportional to

(a) r^2 inside the solenoid and $\frac{1}{r}$ outside

(b) r inside the solenoid and $\frac{1}{r^2}$ outside

(c) r^2 inside the solenoid and $\frac{1}{r^2}$ outside

(d) r inside the solenoid and $\frac{1}{r}$ outside

Q39. In the nuclear shell model, the potential is modeled as $V(r) = \frac{1}{2}m\omega^2r^2 - \lambda\vec{L} \cdot \vec{S}$, $\lambda > 0$. The correct spinparity and isospin assignments for the ground state of ^{13}C is

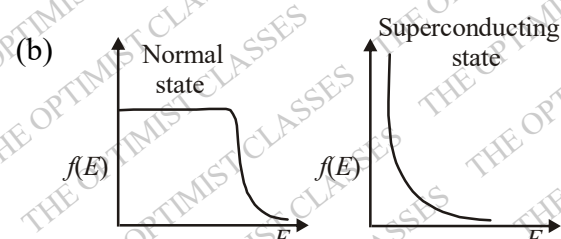
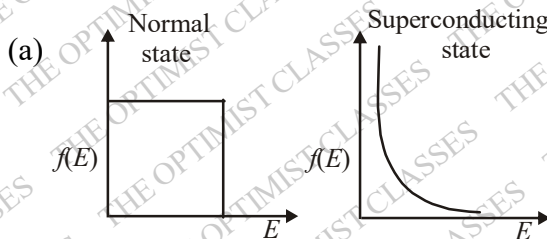
(a) $\frac{1^-}{2}; \frac{-1}{2}$

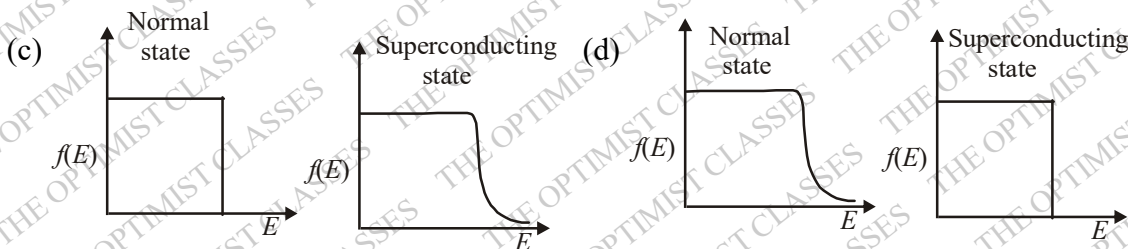
(b) $\frac{1^+}{2}; \frac{-1}{2}$

(c) $\frac{3^+}{2}; \frac{1}{2}$

(d) $\frac{3^-}{2}; \frac{-1}{2}$

Q40. Which of the following represents the electron occupancy for a superconductor in its normal and super-conducting states?





Q41. In a rigid-rotator of mass M , if the energy of the first excited state is $1meV$, then the fourth excited state energy (in meV) is _____.

Q42. The binding energy per molecule of $NaCl$ (lattice parameter is $0.563nm$) is $7.95 eV$, The repulsive term of the potential is of the form $\frac{K}{r^9}$, where K is a constant. The value of the Madelung constant is _____

(upto three decimal places). (Electron charge $e = 1.6 \times 10^{-19} C$; $\epsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$)

Q43. The Hamiltonian for a system of two particles of masses m_1 and m_2 at \vec{r}_1 and \vec{r}_2 having velocities \vec{v}_1 and \vec{v}_2 is

given by $H = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2} \hat{z} \cdot (\vec{r}_1 \times \vec{r}_2)$, where C is a constant. Which one of the following

statements is correct?

- (a) The total energy and total momentum are conserved
- (b) Only the total energy is conserved
- (c) The total energy and the z-component of the total angular momentum are conserved
- (d) The total energy and total angular momentum are conserved

Q44. Given that the Fermi energy of gold is $5.54 eV$, the number density of electrons is _____ $\times 10^{28} m^{-3}$

(upto one decimal place). (Mass of electron = $9.11 \times 10^{-31} kg$; $h = 6.626 \times 10^{-34} Js$; $1eV = 1.6 \times 10^{-19} J$)

Q45. Suppose a linear harmonic oscillator of frequency ω and mass m is in the state $|\psi\rangle = \frac{1}{\sqrt{2}} \left[|\psi_0\rangle + e^{i\frac{\pi}{2}} |\psi_1\rangle \right]$ at

$t=0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle \psi | x | \psi \rangle$ in the

units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t=0$ is _____.

Q46. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If \vec{F}_c and \vec{F}_{Co} denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

- (a) \vec{F}_c is radially outward and $\vec{F}_{Co} = \vec{F}_c$
- (b) \vec{F}_c is radially inward and $\vec{F}_{Co} = -2\vec{F}_c$
- (c) \vec{F}_c is radially outward and $\vec{F}_{Co} = -2\vec{F}_c$
- (d) \vec{F}_c is radially outward and $\vec{F}_{Co} = 2\vec{F}_c$

Q47. A function $y(z)$ satisfies the ordinary differential equation $y'' + \frac{1}{z}y' - \frac{m^2}{z^2}y = 0$, where $m=0, 1, 2, 3, \dots$

Consider the four statements P, Q, R, S as given below:

P : Z^m and Z^{-m} are linearly independent solutions for all values of m .

Q : Z^m and Z^{-m} are linearly independent solutions for all values of $m > 0$.

R : $\ln Z$ and 1 are linearly independent solutions for $m=0$.

S , Z^m and $\ln Z$ are linearly independent solutions for all values of m .

The correct option for the combination of valid statements is

- (a) P , R and S only (b) P and R only (c) Q and R only (d) R and S only

Q48. The average energy U of a one-dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature T is given by

(a) $U = \frac{1}{2} \hbar \omega \coth\left(\frac{1}{2} \beta \hbar \omega\right)$

(b) $U = \frac{1}{2} \hbar \omega \sinh\left(\frac{1}{2} \beta \hbar \omega\right)$

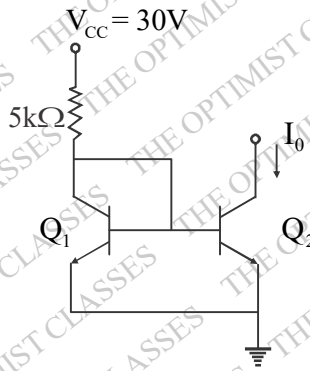
(c) $U = \frac{1}{2} \hbar \omega \tanh\left(\frac{1}{2} \beta \hbar \omega\right)$

(d) $U = \frac{1}{2} \hbar \omega \cosh\left(\frac{1}{2} \beta \hbar \omega\right)$

Q49. Consider a system of eight non-interacting, identical quantum particles of spin $-\frac{3}{2}$ in a one dimensional box of

length L . The minimum excitation energy of the system, in units of $\frac{\pi^2 \hbar^2}{2mL^2}$ is _____

Q50. In the simple current source shown in the figure, Q_1 and Q_2 are identical transistors with current gain $\beta = 100$ and $V_{BE} = 0.7V$



The current I_0 (in mA) is _____ (upto two decimal places)

Q51. The Heaviside function is defined as

$$H(t) = \begin{cases} +1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$$

and its Fourier transform is given by $-2i/\omega$. The Fourier transform of $\frac{1}{2} \left[H\left(t + \frac{1}{2}\right) - H\left(t - \frac{1}{2}\right) \right]$ is

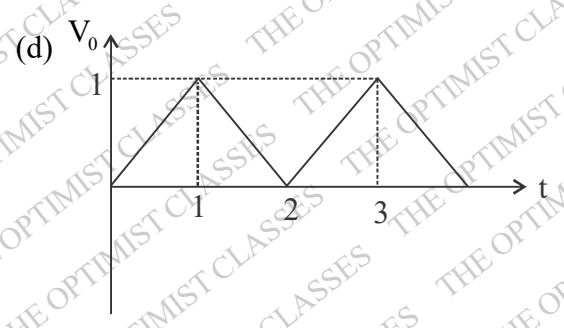
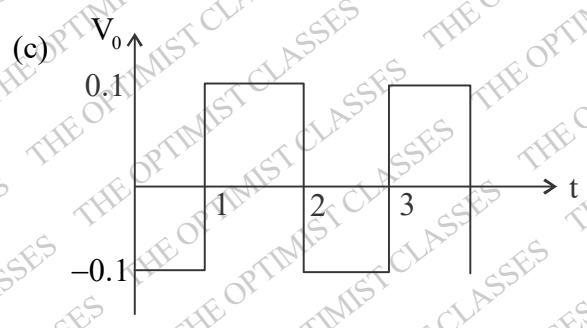
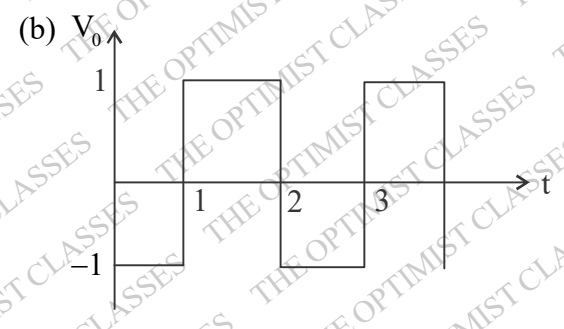
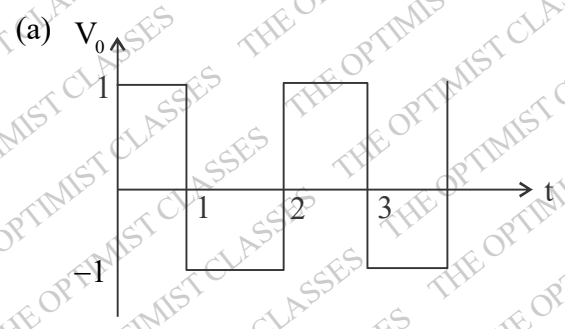
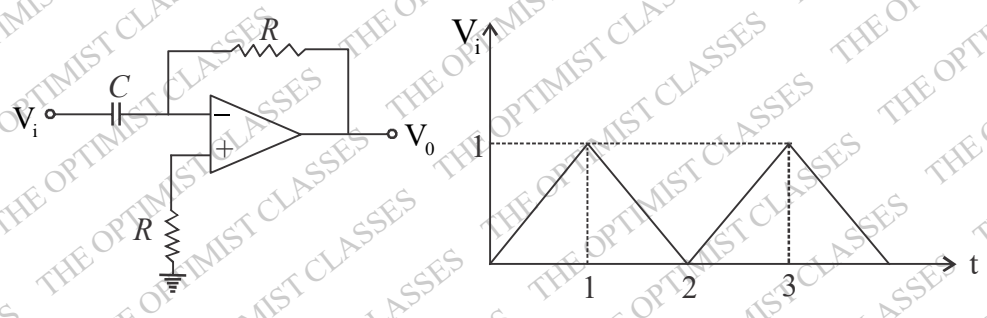
(a) $\frac{\sin \frac{\omega}{2}}{\frac{\omega}{2}}$

(b) $\frac{\cos \frac{\omega}{2}}{\frac{\omega}{2}}$

(c) $\sin \frac{\omega}{2}$

(d) 0

Q52. Consider the circuit shown in the figure, where $RC = 1$. For an input signal V_i shown below, choose the correct V_o from the options:



Q53. Let the Hamiltonian for two spin $-\frac{1}{2}$ particles of equal masses, momenta \vec{p}_1 and \vec{p}_2 and positions \vec{r}_1 and \vec{r}_2 be $H = \frac{1}{2m} p_1^2 + \frac{1}{2m} p_2^2 + \frac{1}{2} m \omega^2 (r_1^2 + r_2^2) + k \vec{\sigma}_1 \cdot \vec{\sigma}_2$ where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar \omega = 0.1 eV$ and $k = 0.2 eV$. If the ground state has net spin zero, then the energy (in eV) is _____

Q54. The excitation wavelength of laser in a Raman effect experiment is 546 nm . If the Stokes line is observed at 552 nm , then the wave number of the anti-Stokes line (in cm^{-1}) is _____

Q55. A monochromatic plane wave (wavelength = 600 nm) $E_0 \exp[i(kz - \omega t)]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$ in the first order of diffraction. Here $E_1 < E_0$ and $\vec{k}_1 = |\vec{k}_1| \left[\frac{1}{2} \hat{x} + \frac{\sqrt{3}}{2} \hat{z} \right]$. The period (in μm) of the diffraction grating is _____ (upto one decimal place).

ANSWER KEY

SECTION - A

- | | | | |
|--------|---------|--------|--------|
| 1. (a) | 2. (c) | 3. (b) | 4. (b) |
| 5. (b) | 6. (c) | 7. (c) | 8. (c) |
| 9. (b) | 10. (a) | | |

SECTION - B

- | | | | |
|----------------------|---------------------|------------------------|----------------------|
| 1. (a) | 2. (0.445 to 0.450) | 3. (c) | 4. (d) |
| 5. (1) | 6. (a) | 7. (d) | 8. (1.3) |
| 9. (-0.67 to -0.66) | 10. (c) | 11. (1844.3 to 1844.5) | |
| 12. (c) | 13. (8) | 14. (d) | 15. (c) |
| 16. (b) | 17. (11) | 18. (b) | 19. (0) |
| 20. (a) | 21. (15) | 22. (b) | 23. (a) |
| 24. (d) | 25. (b) | 26. (d) | 27. (a) |
| 28. (2.830 to 2.835) | 29. (b) | 30. (0.394 to 0.395) | |
| 31. (a) | 32. (6) | 33. (d) | 34. (c) |
| 35. (b) | 36. (4.93 to 4.98) | 37. (b) | 38. (d) |
| 39. (a) | 40. (b) | 41. (10) | 42. (1.745 to 1.751) |
| 43. (c) | 44. (5.9 to 6.0) | 45. (0) | 46. (c) |
| 47. (c) | 48. (a) | 49. (5) | 50. (5.74 to 5.75) |
| 51. (a) | 52. (b) | 53. (-0.3) | 54. (18513 to 18519) |
| 55. (1.2) | | | |