

THE OPTIMIST CLASSES

IIT-JAM TOPPERS



**8
AIR**
MANOJ KUMAR SINGH



**9
AIR**
ABHAY



**12
AIR**
PAWAN



**15
AIR**
SATYAM



**21
AIR**
SOUMIL GIRISH SAHU



**29
AIR**
BHOOMIJA



**31
AIR**
AKSHIT AGGARWAL



**32
AIR**
SHIKHAR CHAMOLI



**33
AIR**
RAVI SINGH ADHIKARI



**44
AIR**
GAURAV JHA



**56
AIR**
SWAPNIL JOSHI



**65
AIR**
LOKESH BHATT



**75
AIR**
GOPESH VISHVAKARMA



**77
AIR**
VAIBHAV

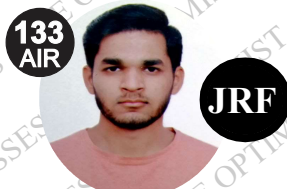


**80
AIR**
SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022



ANNU
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ALANKAR
UP15000162



SAHIL RANA
HR09000108



JAYESTHI
RJ11000161



DASRATH
RJ06000682



VIVEK
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UZAIR AHMED
UP02000246



SURYA PRATAP SINGH
RJ06000232



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VIKAS YADAV
RJ06001102



JYOTSNA KOHLI
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THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

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GATE PAPER 2020

1. Define $[x]$ as the greatest integer less than or equal to x , for each $x \in (-\infty, \infty)$. If $y = [x]$, then area under y for $x \in [1, 4]$ is _____.

- (a) 6 (b) 4
(c) 1 (d) 3

2. He is known for his unscrupulous ways. He always sheds _____ tears to deceive people.

- (a) Crocodile (b) Crocodile's
(c) Fox's (d) Fox

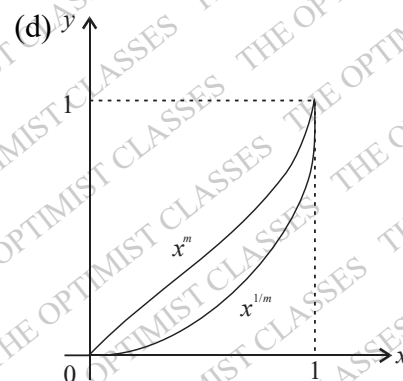
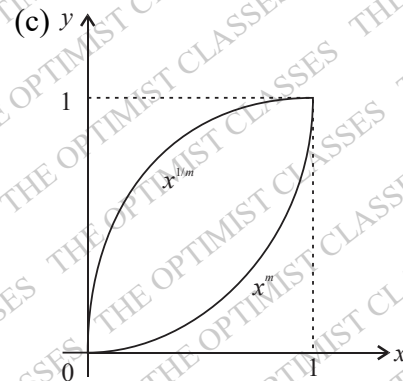
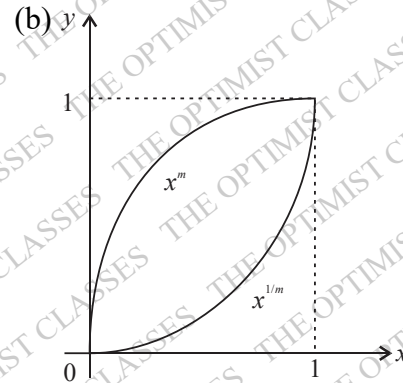
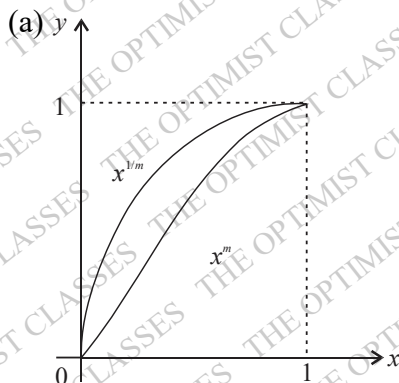
3. Jofra Archer, the England fast bowler, is _____ than accurate.

- (a) Less fast (b) More faster
(c) More fast (d) Faster

4. The sum of the first n terms in the sequence 8, 88, 888, 8888, is _____.

- (a) $\frac{81}{80}(10^n - 1) + \frac{9}{8}n$ (b) $\frac{80}{81}(10^n - 1) + \frac{8}{9}n$
(c) $\frac{81}{80}(10^n - 1) - \frac{9}{8}n$ (d) $\frac{80}{81}(10^n - 1) - \frac{8}{9}n$

5. Select the graph that schematically represents BOTH $y = x^m$ and $y = x^{1/m}$ properly in interval $0 \leq x \leq 1$, for integer values of m , where $m > 1$.



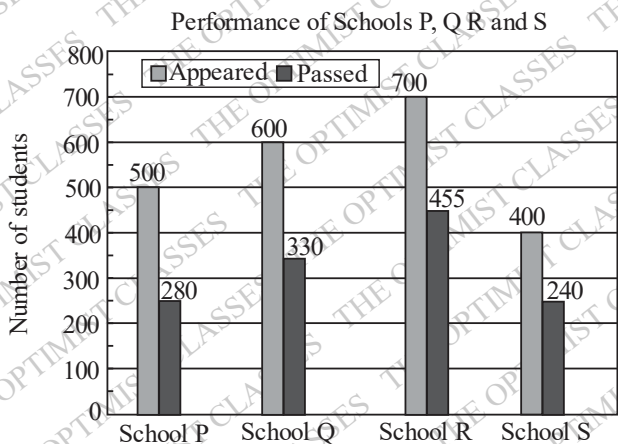
6. Crowd funding deals with mobilisation of funds for a project from a large number of people, who would be willing to invest smaller amounts

through web-based platforms in the project.
 [Based on the above paragraph, which of the following is correct about crowd funding?]
 (a) Funds raised through large contributions on web-based platforms.
 (b) Funds raised through unwilling contributions on web-based platforms.
 (c) Funds raised through voluntary contributions on web-based platforms.
 (d) Funds raised through coerced contributions on web-based platforms.

7. P, Q, R and S are to be uniquely coded using α and β . If P is coded as $\alpha\alpha$ and Q as $\alpha\beta$, then R and S , respectively, can be coded as _____.

- (a) $\beta\alpha$ and $\beta\beta$
- (b) $\alpha\beta$ and $\beta\beta$
- (c) $\beta\beta$ and $\alpha\alpha$
- (d) $\beta\alpha$ and $\alpha\beta$

8. The bar graph shows the data of the students who appeared and passed in an examination for four schools P, Q, R and S . The average of success rates (in percentage) of these four schools is _____.



- (a) 59.3%
- (b) 58.5%
- (c) 59.0%
- (d) 58.8%

9. I do not think you know the case well enough to have opinions. Having said that, I agree with your other point.
 What does the phrase "having said that" mean in the given text?

- (a) Contrary to what I have said
- (b) As opposed to what I have said
- (c) In addition to what I have said
- (d) Despite what I have said

10. Select the word that first the analogy :
 Build : Building :: Grow : _____

- (a) Growed
- (b) Growth

(c) Grown (d) Grew
 11. A particle X is produced in the process $\pi^+ + p \rightarrow K^+ + X$ via the strong interaction. If the quark content of the K^+ is $u\bar{s}$, the quark content of X is

- (a) $u\bar{d}$
- (b) $c\bar{s}$
- (c) uud
- (d) uus

12. A particle is moving in a central force given by $\vec{F} = -\frac{k}{r^3}\hat{r}$, where \hat{r} is the unit vector pointing away from the center of the field. The potential energy of the particle is given by

- (a) $-\frac{k}{r^2}$
- (b) $\frac{k}{r^2}$
- (c) $-\frac{k}{2r^2}$
- (d) $\frac{k}{2r^2}$

13. The total angular momentum j of the ground state of the $^{17}_8\text{O}$ nucleus is

- (a) 1
- (b) $\frac{5}{2}$
- (c) $\frac{1}{2}$
- (d) $\frac{3}{2}$

14. A conducting sphere of radius 1m is placed in air. The maximum number of electrons that can be put on the sphere to avoid electrical breakdown is about 7×10^n , where n is an integer. the value of n is _____.

Assume:

Breakdown electric field strength in air is

$$|\vec{E}| = 3 \times 10^6 \text{ V/m}$$

Permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

[Electron charge $e = 1.60 \times 10^{-19} \text{ C}$]

15. Which one of the following is a universal logic gate?

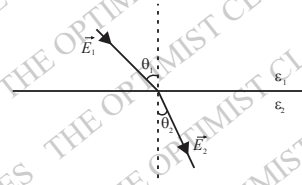
- (a) NOT
- (b) AND
- (c) OR
- (d) NAND

16. A medium ($\epsilon_r > 1, \mu_r = 1, \sigma > 0$) is semi-transparent to an electromagnetic wave when

- (a) Conduction current \ll Displacement current
- (b) Both conduction current and Displacement current are zero
- (c) Conduction current \gg Displacement current

(d) Conduction current = Displacement current

17. Which one of the following relations determines the manner in which the electric field lines are refracted across the interface between two dielectric media having dielectric constants ϵ_1 and ϵ_2 (see figure)?



- (a) $\epsilon_1 \cot \theta_1 = \epsilon_2 \cot \theta_2$
- (b) $\epsilon_1 \sin \theta_1 = \epsilon_2 \sin \theta_2$
- (c) $\epsilon_1 \cos \theta_1 = \epsilon_2 \cos \theta_2$
- (d) $\epsilon_1 \tan \theta_1 = \epsilon_2 \tan \theta_2$

18. Let \hat{a} and \hat{a}^\dagger , respectively denote the lowering and raising operators of a one dimensional simple harmonic oscillator. Let $|n\rangle$ be the energy eigenstate of the simple harmonic oscillator. Given that $|n\rangle$ is also an eigenstate of

$\hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a}$, the corresponding eigenvalue is

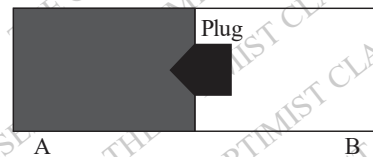
- (a) $n(n+1)$
- (b) n^2
- (c) $(n+1)^2$
- (d) $n(n-1)$

19. Which one of the following is a solution of

$$\frac{d^2 u(x)}{dx^2} = k^2 u(x), \text{ for } k \text{ real?}$$

- (a) $\sin hx$
- (b) $\cos kx$
- (c) $\sin kx$
- (d) e^{-kx}

20. As shown in the figure, an ideal gas is confined to chamber A of an insulated container, with vacuum in chamber B. When the plug in the wall separating the chambers A and B is removed, the gas fills both the chambers. Which one of the following statements is true?



- (a) Internal energy of the gas increases as its atoms have more space to move around
- (b) Temperature of the gas decreases as it expands to fill the space in chamber B

(c) the temperature of the gas remains unchanged

(d) Internal energy of the gas decreases

21. If \vec{E} and \vec{B} are the electric and magnetic fields respectively, then $\vec{E} \cdot \vec{B}$ is

- (a) Odd under parity and even under time reversal
- (b) Even under parity and even under time reversal
- (c) Even under parity and odd under time reversal
- (d) Odd under parity and odd under time reversal

22. A hydrogenic atom is subjected to a strong magnetic field. In the absence of spin orbit coupling, the number of doubly degenerate states created out of d -level is _____.

23. Which one of the following is the correct binary equivalent of the hexadecimal F6C?

- (a) 1100 0110 1111
- (b) 0110 1100 0111
- (c) 1111 0110 1100
- (d) 0110 1111 1100

24. A quantum particle is subjected to the potential

$$V(x) = \begin{cases} \infty, & x \leq -\frac{a}{2} \\ 0, & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & x \geq \frac{a}{2} \end{cases}$$

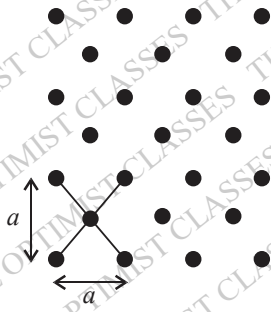
The ground state wave function of the particle is proportional to

- (a) $\cos\left(\frac{\pi x}{2a}\right)$
- (b) $\sin\left(\frac{\pi x}{a}\right)$
- (c) $\cos\left(\frac{\pi x}{a}\right)$
- (d) $\sin\left(\frac{\pi x}{2a}\right)$

25. Choose the correct statement related to the Fermi energy (E_F) and the chemical potential

- (μ) of a metal.
- (a) $\mu > E_F$ at finite temperature
- (b) $\mu < E_F$ at 0 K
- (c) $\mu = E_F$ only at 0 K
- (d) $\mu = E_F$ at finite temperature

26. The number of distinct ways the primitive unit cell can be constructed for the two dimensional lattice as shown in the figure is _____.



27. Particle A with angular momentum $j = \frac{3}{2}$ decays into two particles B and C with angular momenta j_1 and j_2 , respectively.

If $\left| \frac{3}{2}, \frac{3}{2} \right\rangle_A = \alpha \left| 1, 1 \right\rangle_B \otimes \left| \frac{1}{2}, \frac{1}{2} \right\rangle_C$, the value of α is _____.

28. For a complex variable z and the contour $c: |z|=1$ taken in the counter clockwise direction, $\frac{1}{2\pi i} \oint_c \left(z - \frac{2}{z} + \frac{3}{z^2} \right) dz =$ _____.

29. A particle Y undergoes strong decay $Y \rightarrow \pi^- + \pi^-$. The isospin of Y is _____.

30. Far from the Earth, the Earth's magnetic field can be approximated as due to a bar magnet of magnetic pole strength $4 \times 10^{14} Am$. Assume this magnetic field is generated by a current carrying loop encircling the magnetic equator. The current required to do so is about $4 \times 10^n A$, where n is an integer. The value of n is _____ (Earth's circumference : $4 \times 10^7 m$).

31. Consider a diatomic molecule formed by identical atoms. If E_v and E_e represent the energy of the vibrational nuclear motion and respectively, then in terms of the electronic mass m and nuclear mass M , $\frac{E_v}{E_e}$ is proportional to

- (a) $\left(\frac{m}{M} \right)^{3/2}$ (b) $\left(\frac{m}{M} \right)^{1/2}$
 (c) $\frac{m}{M}$ (d) $\left(\frac{m}{M} \right)^2$

32. A real invertible 3×3 matrix M has eigenvalues $\lambda_i, (i=1,2,3)$ and the corresponding eigenvectors are $|e_i\rangle, (i=1,2,3)$ respectively. Which

one of the following is correct ?

- (a) $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i=1,2,3$
 (b) $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$, for $i=1,2,3$
 (c) The eigenvalues of M and M^{-1} are not related.
 (d) $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i=1,2,3$

33. Let p be the momentum conjugate to the generalized coordinate q . If the transformation

$$Q = \sqrt{2}q^m \cos p$$

$$P = \sqrt{2}q^m \sin p$$

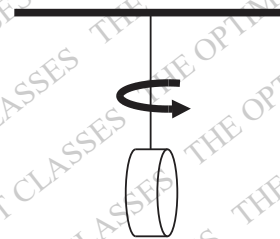
is canonical, then $m =$ _____.

34. If a particle is moving along a sinusoidal curve, the number of degree of freedom of the particle is _____.

35. A small disc is suspended by a fiber such that it is free to rotate about the fiber axis (see figure). For small angular deflections, the Hamiltonian for the disc is given by

$$H = \frac{p_\theta^2}{2I} + \frac{1}{2} \alpha \theta^2,$$

where I is the moment of inertia and α is the restoring torque per unit deflection. The disc is subjected to angular deflections (θ) due to thermal collisions from the surrounding gas at temperature T and p_θ is the momentum conjugate to θ . The average and the root-mean square angular deflection, θ_{avg} and θ_{rms} , respectively are



(a) $\theta_{avg} = 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha} \right)^{3/2}$

(b) $\theta_{avg} \neq 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha} \right)^{1/2}$

(c) $\theta_{avg} \neq 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$

(d) $\theta_{avg} = 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$

36. In the center of mass frame, two protons each having energy 7000 GeV , collide to produce protons and anti-protons. The maximum number of anti-protons produced is _____.
(Assume the proton mass to be $1 \text{ GeV}/c^2$)

37. Consider the Lagrangian

$$L = a \left(\frac{dx}{dt}\right)^2 + b \left(\frac{dy}{dt}\right)^2 + cxy, \text{ where } a, b \text{ and } c$$

are constant. If p_x and p_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is

(a) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$ (b) $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$

(c) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$ (d) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$

38. The Planck's energy density distribution is given by $u(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/k_B T} - 1)}$. At long wavelengths, the energy density of photons in thermal equilibrium with a cavity at temperature T varies as T^α , where α is _____.

39. Let u^μ denote the 4-velocity of a relativistic particle whose square $u^\mu u_\mu = 1$. If $\epsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor then the value of $\epsilon_{\mu\nu\rho\sigma} u^\mu u^\nu u^\rho u^\sigma$ is _____.

40. A sinusoidal voltage of the form $V(t) = V_0 \cos(\omega t)$ is applied across a parallel plate capacitor placed in vacuum. Ignoring the edge effects, the induced *emf* within the region between the capacitor plates can be expressed as a power series in ω . The lowest non-vanishing exponent in ω is _____.

41. According to the Fermi gas model of the nucleus, the nucleons move in a spherical volume of radius $R(= R_0 A^{1/3})$, where A is the mass number and R_0 is an empirical constant with

the dimensions of length). The Fermi energy of the nucleus E_F is proportional to

(a) $\frac{1}{R_0}$ (b) R_0^2

(c) $\frac{1}{R_0^2}$ (d) $\frac{1}{R_0^3}$

42. Consider the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}'$ where

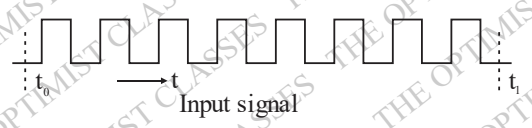
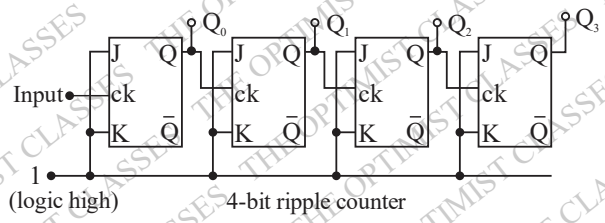
$$\hat{H}_0 = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix} \text{ and } \hat{H}' \text{ is the time inde-}$$

pendent perturbation given by

$$\hat{H}' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}, \text{ where } k > 0. \text{ If, the maxi-}$$

imum energy eigenvalue of \hat{H} is $3eV$ corresponding to $E = 2eV$, the value of k (rounded off to three decimal places) in eV is _____.

43. Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the Q output to the clock input of the following flip-flop (see the figure). The input signal to the counter is a series of square pulses and the change of state is triggered by the falling edge. At time $t = t_0$ the output are in logic low state ($Q_0 = Q_1 = Q_2 = Q_3 = 0$). Then at $t = t_1$, the logic state of the output is



- (a) $Q_0 = 1, Q_1 = 0, Q_2 = 1$ and $Q_3 = 0$
- (b) $Q_0 = 0, Q_1 = 1, Q_2 = 1$ and $Q_3 = 1$
- (c) $Q_0 = 1, Q_1 = 0, Q_2 = 0$ and $Q_3 = 0$
- (d) $Q_0 = 0, Q_1 = 0, Q_2 = 0$ and $Q_3 = 1$

44. Let $|e_1\rangle \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $|e_2\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $|e_3\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Let $S = \{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$. Let R^3 denote the three-dimensional real vector space. Which one of the following is correct?

- (a) S is an orthonormal set
- (b) S is a basis for R^3

(c) $\sum_{i=1}^3 |e_i\rangle |e_i\rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

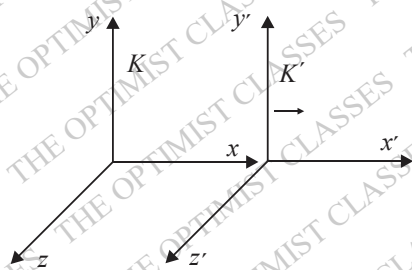
- (d) S is a linearly dependent set

45. A uniform magnetic field $\vec{B} = B_0 \hat{y}$ exists in an inertial frame K . A perfect conducting sphere moves with a constant velocity $\vec{v} = v_0 \hat{x}$ with respect to this inertial frame. The rest frame of the sphere is K' (see figure). The electric and magnetic fields in K and K' are related as

$$\left. \begin{aligned} \vec{E}'_{\parallel} &= \vec{E}_{\parallel} & \vec{E}'_{\perp} &= \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B}) \\ \vec{B}'_{\parallel} &= \vec{B}_{\parallel} & \vec{B}'_{\perp} &= \gamma\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E}\right) \end{aligned} \right\}$$

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

The induced surface charge density on the sphere (to the lowest order in v/c) in the frame K' is



- (a) Uniform over the sphere
- (b) Maximum along z'
- (c) Maximum along y'
- (d) Maximum along x'

46. A free particle of mass M is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the fifth ex-

47. \hat{S}_x denotes the spin operator defined as $\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Which one of the following is correct?

(a) The eigenstates of spin operator \hat{S}_x are

$$|\uparrow\rangle_x \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ and } |\downarrow\rangle_x \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(b) In the spin state $\frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of \hat{S}_x , the probability for obtaining

$$|\uparrow\rangle_x \text{ is } \frac{2 + \sqrt{3}}{4}.$$

(c) The eigenstates of spin operator \hat{S}_x are

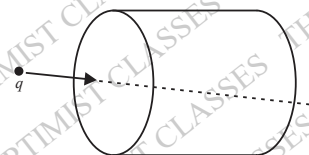
$$|\uparrow\rangle_x \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\downarrow\rangle_x \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

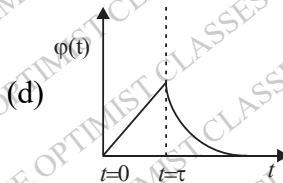
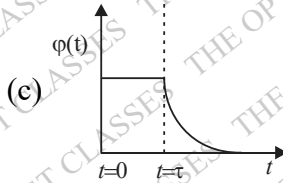
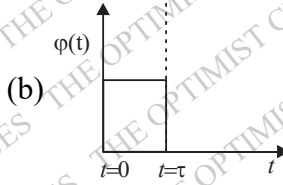
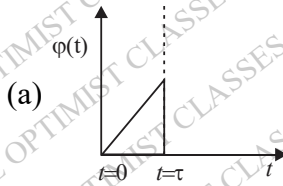
(d) In the spin state $\frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of \hat{S}_x , the probability for obtaining

$$|\uparrow\rangle_x \text{ is } \frac{1}{4}$$

48. If $x = \sum_{k=1}^{\infty} a_k \sin kx$, for $-\pi \leq x \leq \pi$, the value of a_2 is _____.

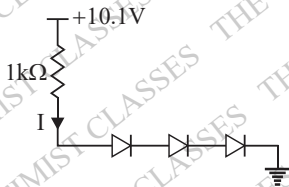
49. A charge q moving with uniform speed enters a cylindrical region in free space at $t = 0$ and exist the region at $t = \tau$ (see figure). Which one of the following options best describes the time dependence of the total electric flux $\phi(t)$, through the entire surface of the cylinder?





50. A plane electromagnetic wave of wavelength λ is incident on a circular loop of conducting wire. The loop radius is a ($a \ll \lambda$). The angle (in degrees), made by the Poynting vector with the normal to the plane of the loop to generate a maximum induced electrical signal, is _____.

51. Consider the circuit given in the figure. Let the forward voltage drop across each diode be $0.7V$. The current I (in mA) through the resistor is _____.



52. Consider a one-dimensional non-magnetic crystal with one atom per unit cell. Assume that the valence electrons (i) do not interact with each other and (ii) interact weakly with the ions. If n is the number of valence electrons per unit cell, then at $0K$,

- (a) The crystal is metallic for even values of n
- (b) The crystal is metallic for odd values of n
- (c) The crystal is non-metallic for any value of n
- (d) The crystal is metallic for any value of n

53. The product of eigenvalues of $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is

- (a) 1
- (b) 2
- (c) 0
- (d) -1

54. Which one of the following matrices does NOT represent a proper rotation in a plane?

- (a) $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$
- (b) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$
- (c) $\begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$
- (d) $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & -\sin \theta \end{pmatrix}$

55. The internal energy U of a system is given by $U(S, V) = \lambda V^{-2/3} S^2$, where λ is a constant of appropriate dimensions; V and S denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system ?

- (a) $\frac{PV^{2/3}}{T} = \text{constant}$
- (b) $\frac{PV}{T^{1/3}} = \text{constant}$
- (c) $\frac{PV^{1/3}}{T^2} = \text{constant}$
- (d) $\frac{P}{V^{1/3}T} = \text{constant}$

56. The radial wave function of a particle in a central potential is given by

$R(r) = A \frac{r}{a} \exp\left(-\frac{r}{2a}\right)$, where A is the normalization constant and a is positive constant of suitable dimensions. If γa is the most probable distance of the particle from the force center, the value of γ is _____.

57. Consider a simple cubic monoatomic Bravais lattice which has a basis with vectors

$\vec{r}_1 = 0, \vec{r}_2 = \frac{a}{4}(\hat{x} + \hat{y} + \hat{z})$, a is the lattice parameter. The Bragg reflection is observed due to the change in the wave vector between the incident and the scattered beam as given by $\vec{K} = n_1 \vec{G}_1 + n_2 \vec{G}_2 + n_3 \vec{G}_3$, where \vec{G}_1, \vec{G}_2 , and \vec{G}_3 are primitive reciprocal lattice vectors. For $n_1 = 3, n_2 = 3$ and $n_3 = 2$, the geometrical structure factor is _____.

58. For a gas of non-interacting particles, the probability that a particle has a speed v in the interval v to $v + dv$ is given by

$$f(v)dv = 4\pi v^2 dv \left(\frac{m}{2\pi k_B T} \right)^{3/2} e^{-mv^2/2k_B T}$$

If E is the energy of a particle, then the maximum in the corresponding energy distribution in units of $E/k_B T$ occurs at _____ (rounded off to one decimal place).

59. An electron in a hydrogen atoms is in the state $n = 3, l = 2, m = -2$. Let \hat{L}_y denote the y -component of the orbital angular momentum operator. If $(\Delta \hat{L}_y)^2 = \alpha \hbar^2$, the value of α is _____.

60. The potential energy of a particle of mass m is given by $U(x) = a \sin(k^2 x - \pi/2)$, $a > 0, k^2 > 0$.

The angular frequency of small oscillations of the particle about $x = 0$ is

- (a) $k^2 \sqrt{\frac{a}{m}}$ (b) $2k^2 \sqrt{\frac{a}{m}}$
 (c) $k^2 \sqrt{\frac{2a}{m}}$ (d) $k^2 \sqrt{\frac{a}{m}}$

61. Consider a gas of hydrogen atoms in the atmosphere of the Sun where the temperature is 5800K. If a sample from this atmosphere contains 6.023×10^{23} of hydrogen atoms in the ground state, the number of hydrogen atoms in the first excited state is approximately 8×10^n , where n is an integer. The value of n is _____. (Boltzmann constant $8.617 \times 10^{-5} eV/K$)

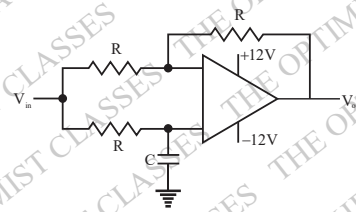
62. Let $f_n(x) = \begin{cases} 0, & x < -\frac{1}{2n} \\ n, & -\frac{1}{2n} < x < \frac{1}{2n} \\ 0, & \frac{1}{2n} \leq x \end{cases}$

The value of $\lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} f_n(x) \sin x dx$ is _____.

63. A hydrogen atom is in an orbital angular momentum state $|l, m = l\rangle$. If \vec{L} lies on a cone which makes a half angle 30° with respect to the z -axis, the value of l is _____.

64. The input voltage (V_{in}) to the circuit shown in

the figure is $2 \cos(100t)V$. The output voltage (V_{out}) is $2 \cos\left(100t - \frac{\pi}{2}\right)V$. If $R = 1k\Omega$, the value of C (in μF) is



- (a) 1 (b) 10
 (c) 100 (d) 0.1

Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed optical branches (n) and acoustic branches (m) due to the lattice vibrations are

- (a) $(n, m) = (4, 2)$ (b) $(n, m) = (2, 4)$
 (c) $(n, m) = (1, 5)$ (d) $(n, m) = (3, 3)$

ANSWER KEY

- | | | | |
|-----|------------|-----|----------------------|
| 1. | (d) | 2. | (c) |
| 3. | (b) | 4. | (d) |
| 5. | (c) | 6. | (c) |
| 7. | (a) | 8. | (c) |
| 9. | (d) | 10. | (c) |
| 11. | (d) | 12. | (c) |
| 13. | (b) | 14. | (14 to 15) |
| 15. | (d) | 16. | (a) |
| 17. | (a) | 18. | (d) |
| 19. | (d) | 20. | (c) |
| 21. | (d) | 22. | (3) |
| 23. | (c) | 24. | (c) |
| 25. | (c) | 26. | (5) |
| 27. | (1) | 28. | (-2) |
| 29. | (2) | 30. | (7) |
| 31. | (b) | 32. | (a) |
| 33. | (0.5) | 34. | (1) |
| 35. | (d) | 36. | (6999) |
| 37. | (a) | 38. | (1) |
| 39. | (0) | 40. | (2) |
| 41. | (c) | 42. | (0.706 to 0.708) |
| 43. | (d) | 44. | (b) |
| 45. | (b) | 46. | (6) |
| 47. | (b) | 48. | (-1) |
| 49. | (b) | 50. | ($\pm 270 \pm 90$) |
| 51. | (8) | 52. | (b) |
| 53. | (d) | 54. | (a) |
| 55. | (c) | 56. | (4) |
| 57. | (2) | 58. | (0.5) |
| 59. | (1) | 60. | (a) |
| 61. | (14 to 15) | 62. | (0) |
| 63. | (3) | 64. | (b) |
| 65. | (a) | | |