THE OPTIMIST CLASSES
IIT-JAM TOPPERS

8 AIR — MANOJ KUMAR SINGH

9 AIR — ABHAY

12 AIR — PAWAN

15 AIR — SATYAM

21 AIR — SOUMIL GIRISH SAHU

29 AIR — BHOOMIJA

31 AIR — AKSHIT AGGARWAL

32 AIR — SHIKHAR CHAMOLI

33 AIR — RAVI SINGH ADHIKARI

44 AIR — GAURAV JHA

56 AIR — SWAPNIL JOSHI

65 AIR — LOKESH BHATT

75 AIR — GOPESH VISHVAKARMA

77 AIR — VAIBHAV

80 AIR — SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022

102 AIR — JRF
ANNU
DL01000308
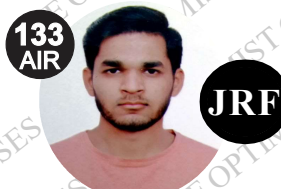
133 AIR — JRF
ALANKAR
UP15000162

155 AIR — JRF
SAHIL RANA
HR09000108

168 AIR — JRF
JAYESTHI
RJ11000161

174 AIR — JRF
DASRATH
RJ06000682

181 AIR — JRF
VIVEK
UK01000439

183 AIR — JRF
UZAIR AHMED
UP02000246

191 AIR — JRF
SURYA PRATAP SINGH
RJ06000232

195 AIR — JRF
HIMANSHU
UP10000095

222 AIR — JRF
CHANDAN
RJ09000159

228 AIR — JRF
SAIKHOM JOHNSON
MN01000196

231 AIR — JRF
AJAY SAINI
RJ06001744

258 AIR — JRF
VIKAS YADAV
RJ06001102

283 AIR — JRF
JYOTSNA KOHLI
UK02000262

318 AIR — JRF
SHYAM SUNDAR
RJ06000615

THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS
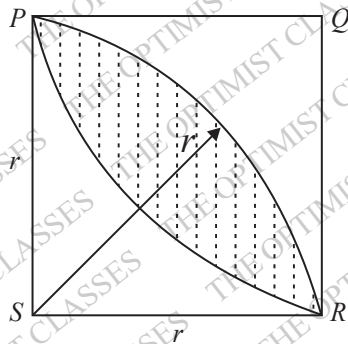
CONTACT: 9871044043

GATE PAPER 2021

1. $\oplus$ and $\odot$ are two operators on numbers p and q such that $p \odot q = p - q$ and $p \oplus q = p \times q$.

   Then, $(9 \odot (6 \oplus 7)) \odot (\oplus (6 \odot 5)) =$

   (a) -40 (b) -26

   (c) 40 (d) -33

2. In the figure shown above, $PQRS$ is a square. The shaded portion is formed by the intersection of sectors of circles with radius equal to the side of the square and centers at S and Q.

   The probability that any point picked randomly within the square falls in the shaded area is

   (a) $\dfrac{1}{2}$ (b) $\dfrac{\pi}{2} - 1$

   (c) $\dfrac{\pi}{4}$ (d) $4 - \dfrac{\pi}{2}$

3. (1) Arun and Aparna are here.

   (2) Arun and Aparna is here.

   (3) Arun's families is here.

   (4) Arun's family is here.

   Which of the above sentences are grammatically CORRECT?

   (a) (3) and (4) (b) (1) and (4)

   (c) (1) and (2) (d) (2) and (4)

4. The author said, "Musicians rehearse before their concerts. Actors rehearse their roles before the opening of a new play. On the other hand. I find it strange that many public speakers think they can just walk on to the stage and start speaking. In my opinion, it is no less important for public speakers to rehearse their talks".

   Based on the above passage, which one of the following is TRUE?

   (a) the author is of the opinion that rehearsing is less important for public speakers than for musicians and actors.

   (b) The author is of the opinion that rehearsel is more important for actors than musicians.

   (c) The author is of the opinion that rehearsing is more important only for musicians than public speakers.

   (d) The author is of the opinion that rehearsing is important for musicians, actors and public speakers.

5. On a planar field, you travelled 3 units East from a point O. Next you travelled 4 units South to arive at point P. Then you travelled from P in the North-East direction such that you arrive at a point that is 6 units East of point O. Net, you travelld in the North-West direction, so that you arrive at point Q that is 8 units North of point P.

   The distance of point Q to point O, in the same units, should be

   (a) 3 (b) 6

   (c) 5 (d) 4

6. 1. Some football players play cricket.

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2. All cricket players play hockey.

Among the options given below, the statement that logically follows from the two statements 1 and 2 above, is:

(a) All hockey players play football.

(b) All football players play hockey.

(c) No football players play hockey.

(d) Some football players play hockey.

7. In an equilateral triangle PQR, side PQ is divided into four equal parts, side QR is divided into six equal parts and side PR is divided into eight equal parts. The length of each subdivided part in cm is an integer. the minimum area of the triangle PQR possible, in cm^2, is

(a) $48\sqrt{3}$ (b) 18

(c) 24   (d) $144\sqrt{3}$

8. Four persons P, Q, R and S are to be seated in a row, R should not be seated at the second position from the left end of the row. The number of distinct seating arrangements possible is

(a) 18   (b) 9

(c) 24   (d) 6

9. Two identical cube shaped dice each with faces numbered 1 to 6 are rolled simultaneously. The probability that an even number is rolled out on each dice is:

(a) 1/8   (b) 1/36

(c) 1/4   (d) 1/12
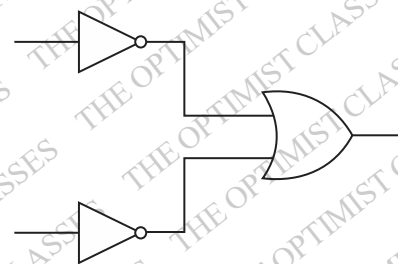
10. The mirror image of the below text about the x-axis is

PHYLAXIS

(a) PH⅄⅃AXIƧ   (b) bH⅄⅂ⱯXIƧ
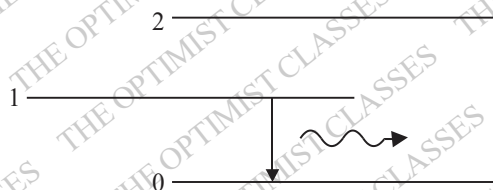
(c) ꟼH⅄⌐ⱯXIƧ   (d) bH⅄⌐ⱯXIS
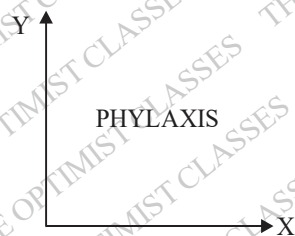
SECTION : PHYSICS

1. The below combination of logic gates represents the operation

(a) AND   (b) NOR

(c) OR   (d) NAND

2. To sustain lasing action in a three-level laser as shown in the figure necessary consition(s) is(are)

(a) lifetime of the energy level 2 should be greater than that of energy level 0.

(b) lifetime of the energy level 1 should be greater than that of energy level 2.

(c) population of the particles in level 1 should be greater than that of level 0.

(d) population of the particles in level 2 should be greater than that of level 1.

3. Consider the potential $U(r)$ defined as:

$$U(r) = -U_0 \frac{e^{-\alpha r}}{r}$$

where α and U_0 are real constants of appropriate dimensions. According to the first Born approximation, the elastic scattering amplitude calculated with $U(r)$ for a (wave-vector) momentum transfer q and $\alpha \to 0$, is proportional to

Useful integral : $\int_0^\infty \sin(qr)e^{-\alpha r}dr = \frac{q}{\alpha^2 + q^2}$

(a) q^{-2} (b) q^2

(c) q^{-1} (d) q

4. Consider a system of three distinguishable particles, each having spin $S = \frac{1}{2}$ such that $S_z = \pm \frac{1}{2}$ with corresponding magnetic moments $\mu_z = \pm\mu$. When the system is placed in an external magnetic field H pointing along the z-axis, the total energy of the system is μH. Let x be the

state where the first spin has $S_z = \frac{1}{2}$. The probability of having the state x and the mean magnetic moment (in the $+z$ direction) of the system in state x are

(a) $\frac{2}{3}, \frac{-2}{3}\mu$ 　　　(b) $\frac{2}{3}, \frac{1}{3}\mu$

(c) $\frac{1}{3}, \frac{2}{3}\mu$ 　　　(d) $\frac{1}{3}, \frac{-1}{3}\mu$

5. Consider a tiny current loop driven by a sinusoidal alternating current. If the surface integral of its time-averaged poynting vector is constant, then the magnitude of the time-averaged magnetic field intensity, at any arbitrary position, $\vec{r}$, is proportional to

(a) $\frac{1}{r^2}$ 　　　(b) r

(c) $\frac{1}{r}$ 　　　(d) $\frac{1}{r^3}$

6. A light source having its intensity peak at the wavelength 289.8nm is calibrated as 10,000K which is the temperature of an equivalent black body radiation. Considering the same calibration, the temperature of light source (in K) having its intensity peak at the wavelength 579.6 nm (rounded off to the nearest integer) is _____.

7. As shown in the figure, X-ray diffraction pattern is obtained from a diatomic chain of atoms P and Q. The diffraction condition is given by $a\cos\theta = n\lambda$, where n is the order of the diffraction peak. Here a is the lattice constant and λ is the wavelength of the X-rays. Assume that atomic form factors and resolution of the instrument do not depend on θ. Then, the intesity of the diffraction peaks is
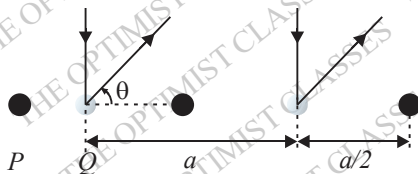
(a) lower for even values of n, when compared to odd values of n.
(b) zero for even values of n.
(c) lower for odd values of n, when compared to even values of n.
(d) zero for odd values of n.

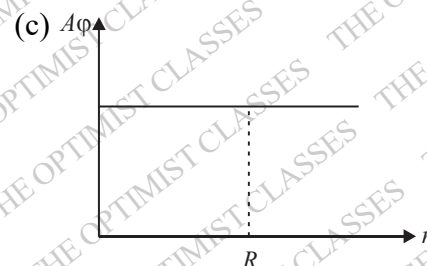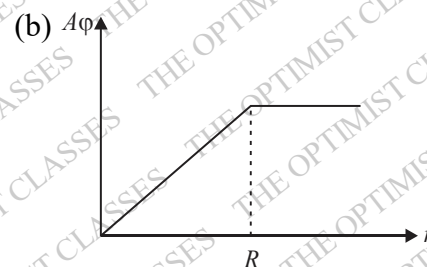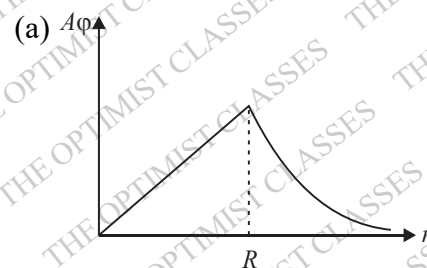8. Consider a solenoid of length L and radius R, where $R \ll L$. A steady current flows through the solenoid. The magnetic field is uniform inside the solenoid and zero outside.
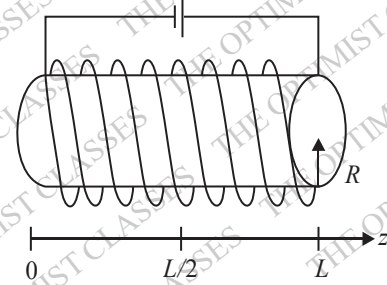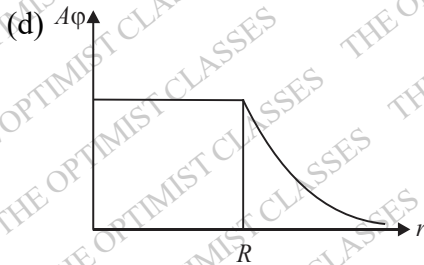

Among the given options, choose the one that best represents the variation in the magnitude of the vector potential $(0, A_\varphi, 0)$ at $z = \frac{L}{2}$, as a function of the radial distance (r) in cylinderical coordinates.

Useful information : The curl of a vector $\vec{F}$, in cylinderical coordinates is

$$\vec{\nabla} \times \vec{F}(r,\varphi,z) = \hat{r}\left[\frac{1}{r}\frac{\partial F_z}{\partial \varphi} - \frac{\partial F_\varphi}{\partial x}\right] + \hat{\varphi}\left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]$$

$$+\hat{z}\frac{1}{r}\left[\frac{\partial(rF_\varphi)}{\partial r} - \frac{\partial F_r}{\partial \varphi}\right]$$

(a)

(b)

(c)

(d)

9. For a finite system of Fermions where the density of states increases with energy, the chemical potential
(a) increases with temperature
(b) decreases with temperature
(c) corresponds to the energy where the occupation probability is 0.5.
(d) does not vary with temperature

10. Consider a point charge $+Q$ of mass m suspended by a massless, inextensible string of length l in free space (permittivity ε_0) as shown in the figure. It is placed at a height $d(d > l)$ over an infinitely large, grounded conducting plane. The gravitational potential energy is assumed to be zero at the position of the conducting plane and is positive above the plane.
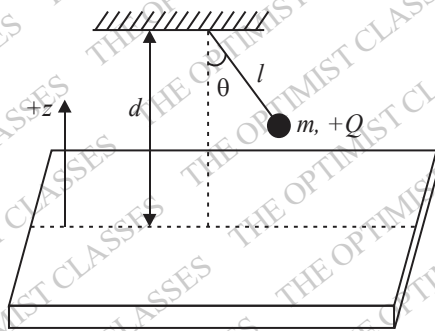

If θ represents the angular position and p_θ its corresponding canonical momentum, then the correct Hamiltonian of the system is

(a) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{8\pi\varepsilon_0(d - l\cos\theta)} + mg(d - l\cos\theta)$

(b) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{8\pi\varepsilon_0(d - l\cos\theta)} - mg(d - l\cos\theta)$

(c) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{16\pi\varepsilon_0(d - l\cos\theta)} + mg(d - l\cos\theta)$

(d) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{16\pi\varepsilon_0(d - l\cos\theta)} - mg(d - l\cos\theta)$

11. Consider the atomic system as shown in the figure, where the Einstein A co-efficients for spontaneous emission for the levels are $A_{2\to1} = 2\times10^7 \sec^{-1}$ and $A_{1\to0} = 10^8 \sec^{-1}$.

If 10^{14} atoms / cm^3 are excited from level 0 to level 2 and a steady state population in level 2 is achieved, then the steady state population at level 1 will be $x \times 10^{13} cm^{-3}$. The value of x (in integer) is
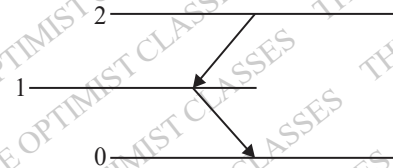
12. If $\vec{a}$ and $\vec{b}$ are constant vectors, $\vec{r}$ and $\vec{p}$ are generalized positions and conjugate momenta, respectively, then for the transformation $Q = \vec{a} \cdot \vec{p}$ and $P = \vec{b} \cdot \vec{r}$ to be canonical, the value of $\vec{a} \cdot \vec{b}$ (in integer) is _____.

13. Consider an atomic gas with number density $n = 10^{20} m^{-3}$, in the ground state at $300K$. The valence electronic configuration of atoms is f^7. The paramagnetic susceptibility of the gas $\chi = m \times 10^{-11}$. The value of m (rounded off to two decimal places) is _____.
(Given : Magnetic permeability of free space $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$
Bohr magneton $\mu_B = 9.274 \times 10^{-24} Am^2$
Boltzmann constant $k_B = 1.3807 \times 10^{-23} JK^{-1}$

14. For the given sets of energy levels of nuclei X and Y whose mass number are odd and even, respectively choose the best suited interpretation.
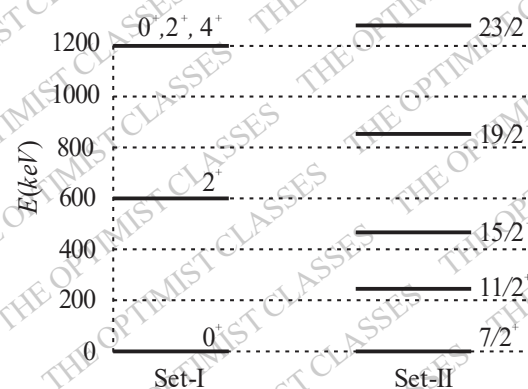

(a) Set-I : Vibrational band of X
Set-II : Rotational band of Y

(b) Set-I : Rotational band of Y
Set-II : Vibrational band of X
(c) Set-I : Rotational band of X
Set-II : Vibrational band of Y
(d) Set-I : Vibrational band of Y
Set-II : rotational band of X

15. As shown in the figure, inverse magnetic suscepti-

bility $\left(\dfrac{1}{\chi}\right)$ is plotted as a function of temperature

(T) for three differential materials in paramagnetic states.
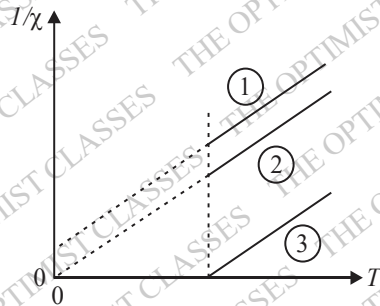
(Curie temperature of ferromagnetic material $= T_C$)
Chose the correct statement from the following.
(a) Material 1 is ferromagnetic ($T < T_C$), 2 is para-magnetic and 3 is antiferromagnetic ($T < T_N$).
(b) Material 1 is paramagnetic, 2 is antiferromag-netic ($T < T_N$) and 3 is ferromagnetic ($T < T_C$).
(c) Material 1 ferromagnetic ($T < T_C$), 2 is antifer-romagnetic ($T < T_N$), and 3 is paramagnetic.
(d) Material 1 is antiferromagnetic ($T < T_N$), 2 is paramagnetic, and 3 is ferromagnetic ($T < T_C$).

16. The normalized radial wave function of the sec-ond excited state of Hyrogen atom is

$$R(r) = \frac{1}{\sqrt{24}}(a^{-3/2})\frac{r}{a}(e^{-r/2a})$$

where a is the Bohr radius r is the distance from the centre of the atom. The distance at which the electron is most likely to be found is $y \times a$. The value of y (in integer) is _____.

17. Consider a cross-section of an electromagnet hav-ing an air-gap of $5cm$ as shown in the figure. It consists of a magnetic material $(\mu = 20000\mu_0)$ and is driven by a coil having $NI = 10^4 A$, where N is the number of turns and I is the current in Ampere
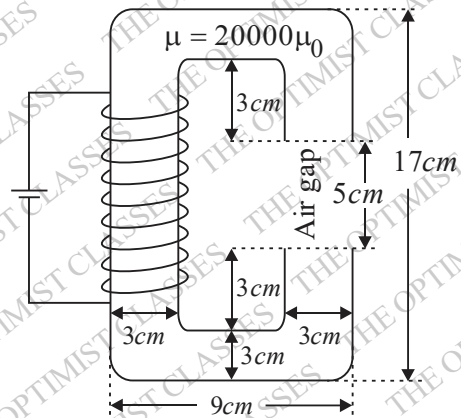

Ignoring the fringe fields, the magnitude of the magnetic field $\vec{B}$ (in Tesla, rounded off to two decimal places) in the air-gap between the mag-netic poles is _____

18. A system of two atoms can be in three quantum state having energies $0, \varepsilon$ and 2ε. The system is in equillibrium at temperature $T = (k_B\beta)^{-1}$. Match the following statistics with the partition function.

Statistics	Partition function
CD: Classical (distinguishable particles)	Z1: $e^{-\beta\varepsilon} + e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon}$
CI: Classical (indistinguishable particles)	Z2: $1 + e^{-\beta\varepsilon} + 2e^{-\beta\varepsilon} + e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}$
FD: Fermi Dirac	Z3: $1 + 2e^{-\beta\varepsilon} + 3e^{-2\beta\varepsilon} + 2e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}$
BE: Bose -Einstein	Z4: $\dfrac{1}{2} + e^{-\beta\varepsilon} + \dfrac{3}{2}e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon} + \dfrac{1}{2}e^{-4\beta\varepsilon}$

(a) $CD : Z1, \ CI : Z2, \ FD : Z3, \ BE : Z4$
(b) $CD : Z2, \ CI : Z3, \ FD : Z4, \ BE : Z1$
(c) $CD : Z4, \ CI : Z1, \ FD : Z2, \ BE : Z3$
(d) $CD : Z3, \ CI : Z4, \ FD : Z1, \ BE : Z2$

19. Choose the graph that best describes the varia-tion of dielectric constant (ε_r) with temperature (T) in a ferro-electric material. (T_C is the Curie temperature)

(a) ε_r

(b) ε_r

(c) ε_r

(d) ε_r
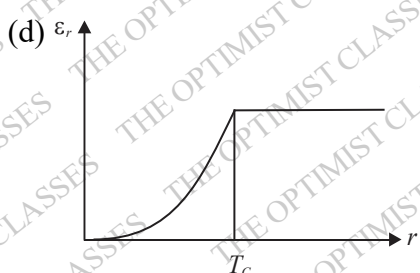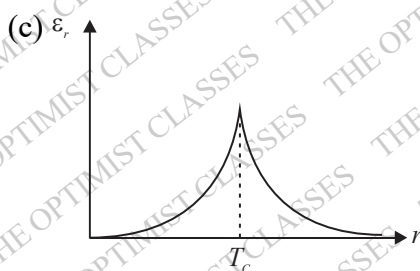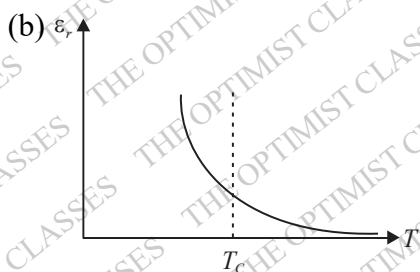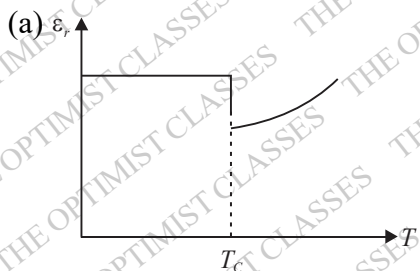
x, then acceptable plot(s) of $\psi_n(x) = e^{-x/2} y_n(x)$ is(are)

(a) $n=1$

(b) $n=0$

(c) $n=1$

(d) $n=0$
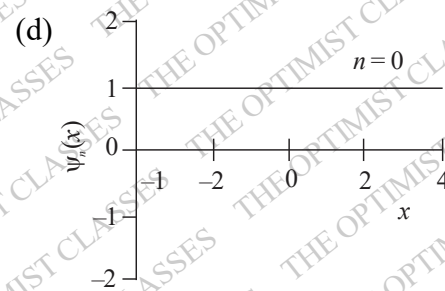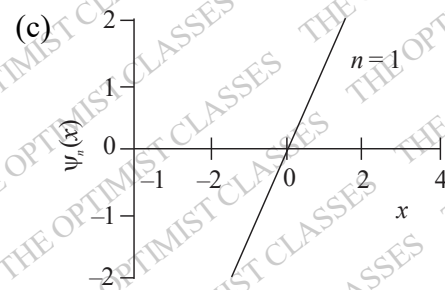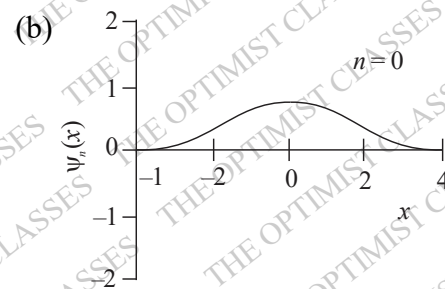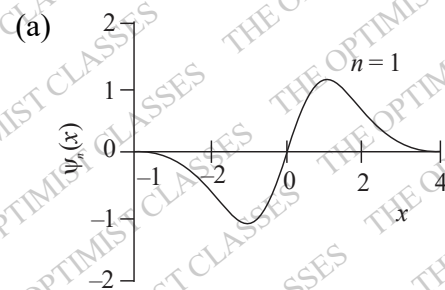
20. The spin-orbit effect splits the $^2P \to{}^2 S$ transition (wavelength, $\lambda = 6252 \mathring{A}$) in Lithium into two lines with separation of $\Delta\lambda = 0.14\mathring{A}$. The corresponding positive value of energy difference between the above tow lines, in eV , is $m\times10^{-5}$. The value of m (rounded off to the nearest integer) is _____ .

[Given: Planck's constant, $h = 4.125\times10^{-15}eV - \sec$, speed of light, $c = 3\times10^8\,m\sec^{-1}$].

21. If $y_n(x)$ is a solution of the differential equation $y'' - 2xy' - 2ny = 0$ where n is an integer and the prime (') denotes differentiation with rrespect to

22. Consider a spin $S = \dfrac{\hbar}{2}$ particle in the state $|\phi\rangle = \dfrac{1}{3}\begin{bmatrix} 2+i \\ 2 \end{bmatrix}$. The probability that a measurement finds the state with $S_x = +\dfrac{\hbar}{2}$ is

(a) $\dfrac{17}{18}$ (b) $\dfrac{5}{18}$

(c) $\dfrac{15}{18}$ (d) $\dfrac{11}{18}$

23. Consider a state described by $\psi(x,t) = \psi_2(x,t) + \psi_4(x,t)$, where $\psi_2(x,t)$ and $\psi_4(x,t)$ are respectively the second and fourth normalized harmonic oscillator wave functions and ω is the angular frequency of the harmonic oscillator. The wave function $\psi(x,t) = 0$ will be orthogonal to $\psi(x,t)$ at time t equal to

(a) $\dfrac{\pi}{2\omega}$ (b) $\dfrac{\pi}{4\omega}$

(c) $\dfrac{\pi}{6\omega}$ (d) $\dfrac{\pi}{\omega}$

24. Choose the correct statement from the following.
(a) Gallium Arsenide is an indirect band gap semiconductor
(b) Conductivity of semiconductor decreases with increase in temperature
(c) Silicon is a direct band gap semiconductor
(d) Conductivity of metals decreases with increase in temperature

25. A two-dimensional square lattice has lattice constant a.
k represents the wavevector in reciprocal space. The coordenates (k_x, k_y) of reciprocal space where band gap(s) can occur, are

(a) $\left(\pm\dfrac{\pi}{a}, \pm\dfrac{\pi}{1.3a}\right)$ (b) $\left(\pm\dfrac{\pi}{a}, \pm\dfrac{\pi}{a}\right)$

(c) $\left(\pm\dfrac{\pi}{3a}, \pm\dfrac{\pi}{a}\right)$ (d) $(0,0)$

26. A matter wave is represented by the wave function: $\psi(x,y,z,t) = A e^{i(4x+3y+5z-10\pi t)}$

where A is a constant. The unit vector repersenting the direction of propagation of this matter wave is

(a) $\dfrac{4}{5\sqrt{2}}\hat{x} + \dfrac{3}{5\sqrt{2}}\hat{y} + \dfrac{1}{\sqrt{2}}\hat{z}$

(b) $\dfrac{1}{5\sqrt{2}}\hat{x} + \dfrac{3}{5\sqrt{2}}\hat{y} + \dfrac{1}{\sqrt{2}}\hat{z}$

(c) $\dfrac{3}{5\sqrt{2}}\hat{x} + \dfrac{4}{5\sqrt{2}}\hat{y} + \dfrac{1}{5\sqrt{2}}\hat{z}$

(d) $\dfrac{1}{5\sqrt{2}}\hat{x} + \dfrac{4}{5\sqrt{2}}\hat{y} + \dfrac{3}{5\sqrt{2}}\hat{z}$

27. A material is placed in a magnetic field intensity $\vec{H}$. As a result, bound current density $\vec{J}_b$ is induced and magnetization of the material is $\vec{M}$. The magnetic flux density is $\vec{B}$. Choose the correct options(s) valid at the surface of the material.
(a) $\vec{\nabla}.\vec{H} = 0$ (b) $\vec{\nabla}.\vec{B} = 0$
(c) $\vec{\nabla}.\vec{J}_b = 0$ (d) $\vec{\nabla}.\vec{M} = 0$

28. Assume that $^{13}N\,(Z = 7)$ undergoes first forbidden β^+ decay from its ground state with spin-partiy J_i^π, to a final state J_f^π. The possible values for J_i^π and J_f^π, respectively, are

(a) $\dfrac{1}{2}^-, \dfrac{1}{2}^-$ (b) $\dfrac{1}{2}^-, \dfrac{5}{2}^+$

(c) $\dfrac{1}{2}^+, \dfrac{5}{2}^+$ (d) $\dfrac{1}{2}^+, \dfrac{1}{2}^-$

29. A uniform block of mass M slides on a smooth horizontal bar. Another mass m is connected to it by an inextensible string of length l of negligible mass, and is constrained to oscillate in the X-Y plane only. Neglect the sizes of the masses. The number of degrees of freedom of the system is two and the generalized coordinates are chosen as x and θ as shown in the figure.
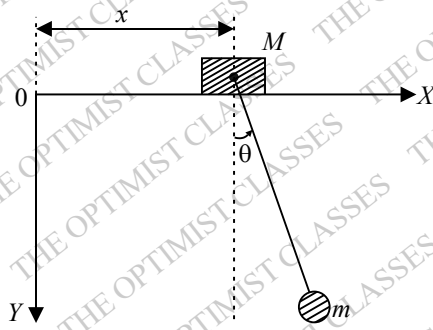
If p_x and p_θ are the generalized momenta corresponding to x and θ, respectively, then the correct option(s) is(are)

(a) p_θ is conserved

(b) $p_\theta = ml^2\dot{\theta} - ml\cos\theta\dot{x}$

(c) p_x is conserved

(d) $p_x = (m+M)\dot{x} + ml\cos\theta\dot{\theta}$

30. In a semiconductor, the ratio of the effective mass of hole to electron is $2:11$ and the ratio of average realxation time for hole to electron is $1:2$. The ratio of the mobility of the hole to electron is

(a) $4:9$ (b) $11:4$

(c) $4:11$ (d) $9:4$

31. The free energy of a ferromagnet is given by

$F = F_0 + a_0(T - T_C)M^2 + bM^4$, where F_0, a_0 and b are positive constants, M is the magnetization, T is the temperature, and T_C is the Curie temperature. The relation between M^2 and T is best depicted by

(a)

(b)

(c)

(d)
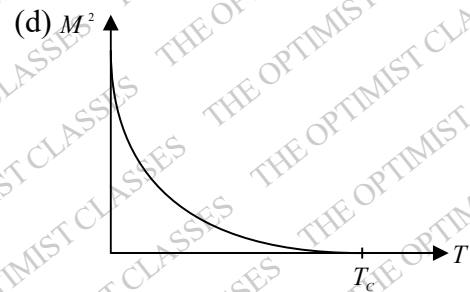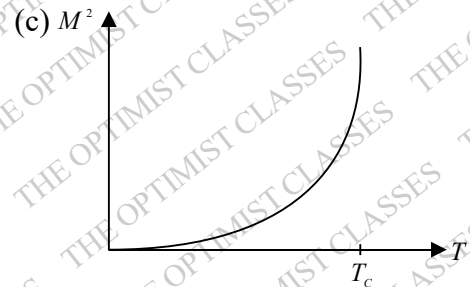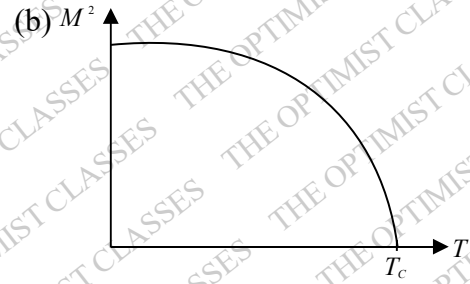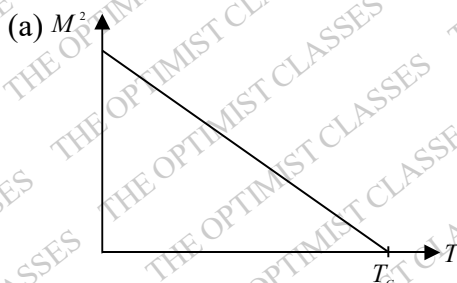
32. As shown in the figure, an electromagnetic wave with intensity I_I is incident at the interface of two media having refractive indices $n_1 = 1$ and $n_2 = \sqrt{3}$. The wave is reflacted with intensity I_R and transmitted with intensity I_T. Permeability of each medium is the same. (Reflection coefficient $R = \dfrac{I_R}{I_I}$ and Transmission cofficient $T = \dfrac{I_T}{I_I}$).
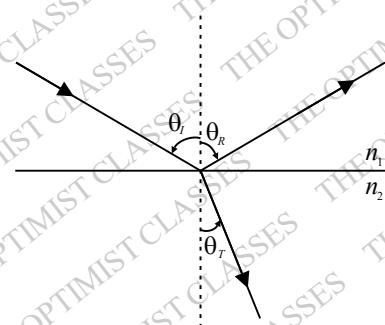
Choose the correct statement(s).

(a) $T = 1$ if $\theta_I = 60°$ and polarization of incident

light is perpendicular to the plane of incidence

(b) $R = 0$ if $\theta_I = 60°$ and polarization of incident light is perpendicualr to the plane of incidence

(c) $R = 0$ if $\theta_I = 0°$ and polarization of incident light is parallel to the plane of incidence

(d) $T = 1$ if $\theta_I = 60°$ and polarization of incident light is parallel to the plane of incidence

33. The transition line, as shown in the figure, arises between $^2D_{3/2}$ and $^2P_{1/2}$ states without any external magnetic field. The number of lines that will appear in the presence of a weak magnetic field (in integer) is_____.

34. Consider a single one-dimensional harmonic oscillator of angular frequency ω, in equilibrium at temperature $T = (k_B\beta)^{-1}$. The states of the harmonic oscillator are all non-degenerate having energy $E_n = \left(n + \dfrac{1}{2}\right)\hbar\omega$ with equal probability, where n is the quantum number. The Helmholtz free energy of the oscillator is
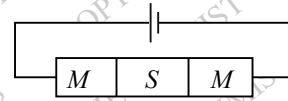
(a) $\dfrac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 - \exp(-\beta\hbar\omega)\right]$

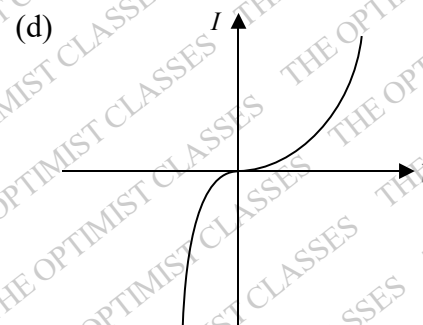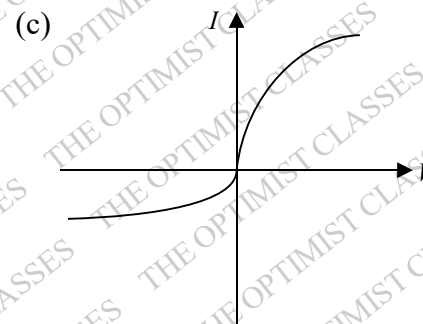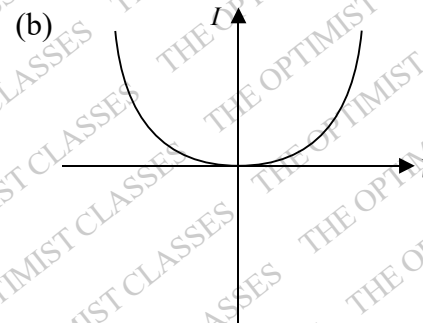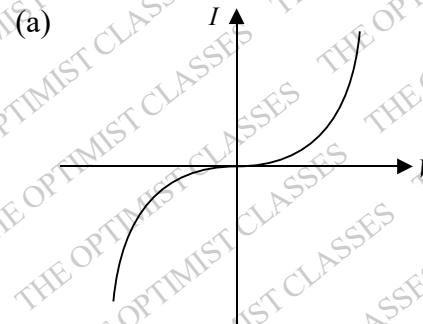(b) $\dfrac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 + \exp(-\beta\hbar\omega)\right]$

(c) $\dfrac{\hbar\omega}{2} + \beta^{-1}\ln\left[1 - \exp(\beta\hbar\omega)\right]$

(d) $\beta^{-1}\ln\left[1 - \exp(-\beta\hbar\omega)\right]$

35. As shown in the figure, two metal-semiconductor junctions are formed between an n-type semiconducot S and metal M. The work functions of S and M are φ_S and φ_M, respectively with $\varphi_M > \varphi_S$.

The 1-V characteristics (on linear scale) of the junctions is best represented by

(a)

(b)

(c)

(d)

36. Two observers O and O' observe two events P and Q. The observers have a constant relative speed of $0.5c$. In the units, where the speed of
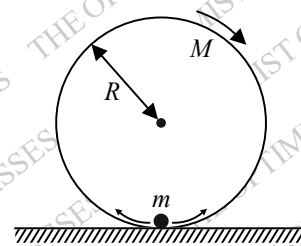
light, c, is taken as unity, the observer O obtained the following coordinates:

Event P: $x = 5$, $y = 3$, $z = 5$, $t = 3$

Event Q: $x = 5$, $y = 1$, $z = 3$, $t = 5$

The length of the space-time interval between these two events, as measured by O', is L. The value of $|L|$ (in integer is____.

37. P and Q are two Hermitian matrices and there exists a matrix R, which diagonalizes both of them, such that $RPR^{-1} = S_1$ and $RQR^{-1} = S_2$, where S_1 and S_2 are diagonal matrices. The correct statement(s) is/are

(a) The matrix QP can have complex eigenvalues

(b) The matrices P and Q commute

(c) All the elements of both matrices S_1 and S_2 are real

(d) The matrix PQ can have complex eigenvalues

38. A linear charged particle accelerator is driven by an alternating voltage source operating at $10MHz$. Assume that it is used to accelerate electrons. After a few drift-tubes, the electrons attain a velocity $2.9 \times 10^8 m/s$. The minimum length of each drift-tube, in m, to accelerate the electrons further (rounded off to one decimal place) is____.

39. The Coulomb energy component in the binding energy of a nucleus is $18.432 MeV$. If the radius of the uniform and spherical charge distribution in the nucleus is $3fm$, the correspoinding atomic number (rounded off to the nearest integer) is____.

$$\left[\text{Given:} \frac{e^2}{4\pi\varepsilon_0} = 1.44 MeV\ fm \right]$$

40. A hoop of mass M and radius R rolls without slipping along a straight line on a horizontal surface as shown in the figure. A point mass m slides without friction along the inner surface of the hoop, performing small oscillations about the mean position. The number of degrees of freedom of the system (in integer) is____.

41. The time derivative of a differentiable function $g(q_i, t)$ is added to a Lagrangian $L(q_i, \dot{q}_i, t)$ such that

$$L' = L(q_i, \dot{q}_i, t) \frac{d}{dt} g(q_i, t)$$

where $q_i, \dot{q}_i, t$ are the generalized coordinates, generalized velocitites and time, respectively. Let p_i be the generalized momentum and H the Hamiltonian associated with $L(q_i, \dot{q}_i, t)$. If p_i and H' are those associated with L', then the correct option(s) is/are:

(a) Both L and L' satisfy the Euler-Lagrange's equations of motion

(b) If p_i is conserved, then p_i is necessarily conserved
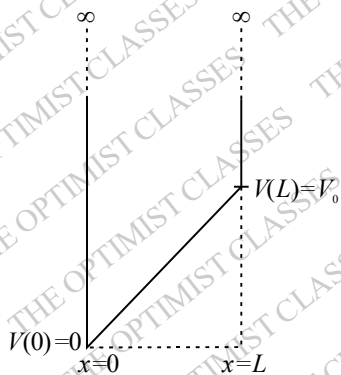
(c) $p_i = p_i + \frac{\partial}{\partial q_i} g(q_i, t)$

(d) $H' = H + \frac{d}{dt} g(q_i, t)$

42. Among the term symbols

$^4S_1$, $^2D_{7/2}$, $^3S_1$ and $^2D_{5/2}$

Choose the option(s) possible in the LS coupling notation.

(a) $^2D_{7/2}$ (b) $^4S_1$

(c) $^3S_1$ (d) $^2D_{5/2}$

43. Consider a particle in a one-dimensional infinite potential well with its walls at $x = 0$ and $x = L$. The system is perturbed as shown in the figure.

The first order correction to the energy eigenvalue is

(a) $\dfrac{V_0}{4}$ (b) $\dfrac{V_0}{2}$

(c) $\dfrac{V_0}{5}$ (d) $\dfrac{V_0}{3}$

44. An electromagnetic wave having field $\vec{E} = 8\cos(kz - \omega t)\hat{y} \, Vcm^{-1}$ is incident at $90°$ (normal incidence) on a square slab from vacuum (with refractive index $n_0 = 1.0$) as shown in the figure. The slab is composed of two different material with refractive indices n_1 and n_2. Assume that the permeability of each mecdium is the same. After passing through the slab for the first time, the electric field amplitude, in Vcm^{-1}, of the electromagnetic wave, which emerges from the slab in region 2, is closest to
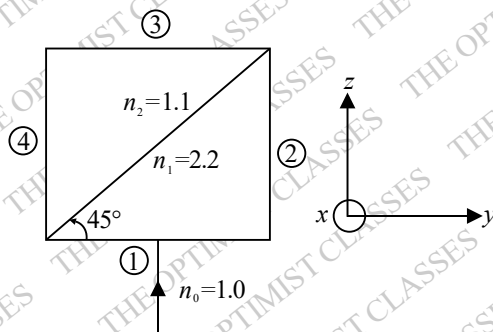
(a) $\dfrac{11}{3.2}$ (b) $\dfrac{11}{25.6}$

(c) $\dfrac{11}{13.8}$ (d) $\dfrac{11}{1.6}$

45. For a two-nucleon system in spin singlet state, the spin is represented through the Pauli matrices σ_1, σ_2 for particles 1 and 2, respectively. The value of $(\sigma_1.\sigma_2)$ (in integer) is _____.

46. In an experiment, it is seen that an electric-dipole $(E1)$ transition can connect an initial nuclear state of spin-partiy $J_i^\pi = 2^+$ to final state J_f^π. All possible values of J_f^π are

(a) $1^+, 2^+, 3^+$ (b) $1^-, 2^-$

(c) $1^-, 2^-, 3^-$ (d) $1^+, 2^+$

47. Three non-interacting bosonic particles of mass m each, are in a one-dimensional infinite potential well of width a. The energy of the third excited state of the system is $x \times \dfrac{\hbar^2 \pi^2}{ma^2}$. The value of x (in integer) is _____.

48. The spaing between two consecutive S-branch lines of the rotational Raman spectra of hydrogen gas is $243.2 cm^{-1}$. After excitation with a laser of a wavelength $514.5 nm$, the stoke's line appeared at $17611.4 cm^{-1}$ for a particular energy level. The wave number (rounded off to the nearest integer), in cm^{-1}, at which stoke's line with appear for the next highe energy level is _____.

49. A contour integral is defined as:
$$I_n = \oint_C \frac{dz}{(z - n)^2 + \pi^2}$$

Where n is a positive integer C is the closed cotour, as shown in the figure, consisting of the line from -100 to 100 and the smicircletraversed in the counter-clockwise sense.
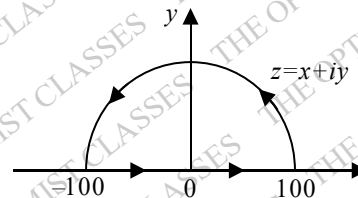
The value of $\sum_{n=1}^{5} I_n$ (in integer) is _____.

50. Consider two concentric conducting spherical shells as shown in the figure. The inner shell has a radius 'a' and carries a charge $+Q$. The outer shell has a radius 'b' and carries a charge $-Q$. The empty

The space between them is half-filled by a hemispherical shell of a dielectric having permittivity ε_1. The remaining space between the shells is filled with air having the permittivity ε_0.
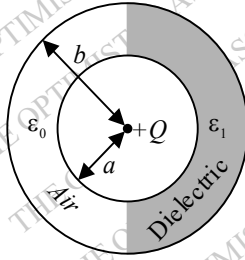
The electric field at a radial distance r from the center and between the shells $(a < r < b)$ is

(a) $\dfrac{q}{4\pi\varepsilon_0}\dfrac{\hat{r}}{r^2}$ on the air side and $\dfrac{q}{4\pi\varepsilon_1}\dfrac{\hat{r}}{r^2}$ on the dielectric side

(b) $\dfrac{Q}{2\pi(\varepsilon_0+\varepsilon_1)}\dfrac{\hat{r}}{r^2}$ every where.

(c) $\dfrac{Q}{2\pi\varepsilon_0}\dfrac{\hat{r}}{r^2}$ on the air side and $\dfrac{Q}{2\pi\varepsilon_1}\dfrac{\hat{r}}{r^2}$ on the dielectric side.

(d) $\dfrac{Q}{4\pi(\varepsilon_0+\varepsilon_1)}\dfrac{\hat{r}}{r^2}$ every where

51. A function $f(t)$ is defined only for $t \ge 0$. The Laplace transform of $f(t)$ is

$$L(f;s)=\int_0^\infty e^{-st}f(t)dt$$

whereas the fourier transform of $f(t)$ is

$$\tilde{f}(\omega)=\int_0^\infty f(t)e^{-i\omega t}dt$$

The correct statement (s) is/are:

(a) $L(f;s)$ and $\tilde{f}(\omega)$ can be made connected.

(b) The variable s is always real.

(c) The variable s can be complex.

(d) $L(f;s)$ and $\tilde{f}(\omega)$ can never be made connected.

52. The spin $\vec{S}$ and orbital angular momentum $\vec{L}$ of an atom precess about $\vec{J}$, the total angular momentum. $\vec{J}$ precesses about an axis fixed by a magnetic field $\vec{B}_1 = 2B_0\hat{z}$, where B_0 is a constant. Now the magnetic field is changed to $\vec{B}_2 = B_0\left(\hat{x}+\sqrt{2}\hat{y}+\hat{z}\right)$. Given the orbital angular momentum quantum number $l = 2$ and spin quan-

tum number $s=\dfrac{1}{2}$, θ is the angle between $\vec{B}_1$ and $\vec{J}$ for the largest possible values of total angular quantum number j and its z-component j_z. The value of θ (in degree, rounded off to the nearest integer is _____.

53. The donor concentration in a sample of n-type silicon is increased by a factor of 100. Assuming the sample to be non-degenerate, shift in the fermilevel (in meV) at $300K$ (rounded off to the nearest integer) is _____.
[Given $k_BT = 25\ meV$ at $300K$]

54. The Gell-Mann-Okuba mass formula defines the mass of baryons as $M = M_0 + aY + b\left[I(I+1)\right.$ $\left.-\dfrac{1}{4}Y^2\right]$, where M_0, a and b are constants. I represents the isospin and y represents the hypercharge. If the mass of Σ hyperons is same as that of Λ hyperons, then the correct option (s) is (are)
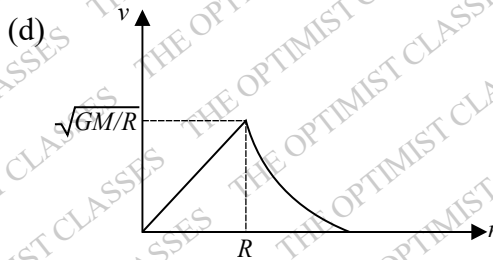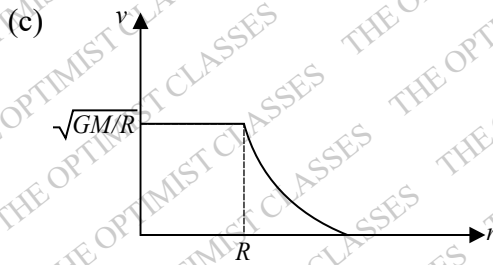
(a) $M \propto Y$

(b) $M \propto I(I+1)$

(c) M does not depend on I

(d) M does not depend on Y

55. Consider a spherical galaxy of total mass M and radisu R, having a uniform matter distribtuion. In this idealized situation, the orbital speed v of a star of mass $m(m << M)$ as a function of the distance r from the galactic centre is best described by (G is the universal gravitational constant)

(a)

(b)

(c)

(d)
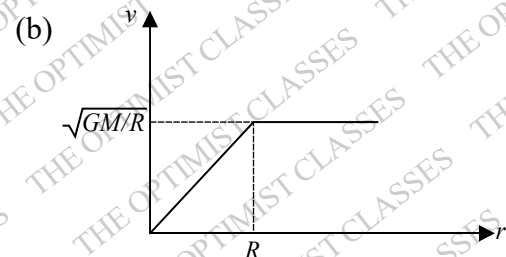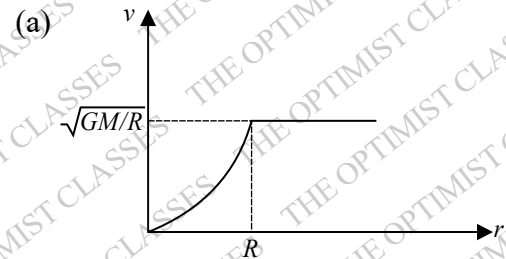
ANSWER KEY

1.	(c)	2.	(b)
3.	(b)	4.	(d)
5.	(c)	6.	(a)
7.	(d)	8.	(a)
9.	(c)	10.	(b)

SECTION - PHYSICS

1.	(d)	2.	(b, c)
3.	(a)	4.	(d)
5.	(c)	6.	(5000)
7.	(c)	8.	(a)
9.	(b, c)	10.	(c)
11.	(2)	12.	(–1)
13.	(5.40 to 5.50)	14.	(d)
15.	(d)	16.	(4)
17.	(0.24 to 0.26)	18.	(c)
19.	(c)	20.	(3 to 5)
21.	(a, b)	22.	(a)
23.	(a)	24.	(b)
25.	(a, b, c)	26.	(a)
27.	(b, c)	28.	(b)
29.	(c, d)	30.	(b)
31.	(a)	32.	(d)
33.	(6)	34.	(a)
35.	(a)	36.	(2)
37.	(b, c)	38.	(14.0 to 15.0)
39.	(8 to 9)	40.	(2)
41.	(a, c)	42.	(c, d)
43.	(b)	44.	(d)
45.	(–3)	46.	(c)
47.	(5 to 6)	48.	(17367 to 17371)
49.	(5)	50.	(b)
51.	(a, c)	52.	(27 to 93)
53.	(115 to 116)	54.	(a, c)
55.	(d)		