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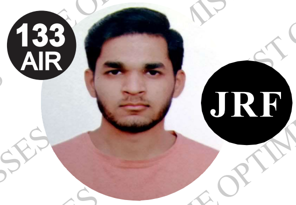

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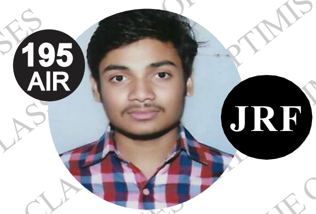

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CSIR-UGC-NET/JRF-JUNE - 2011

PREVIOUS YEAR QUESTION

PHYSICAL SCIENCES

PART - B

21. A particle of unit mass moves in potential $V(x) = ax^2 + \frac{b}{x^2}$ where a and b are positive constants

The angular frequency of small oscillations about the minimum of the potential is:

(a) $\sqrt{8b}$ (b) $\sqrt{8a}$ (c) $\sqrt{8a/b}$ (d) $\sqrt{8b/a}$

22. A signal of frequency 10kHz is being digitized by an A/D converter. A possible sampling time which can be used is:

(a) 100 μs (b) 40 μs (c) 60 μs (d) 200 μs

23. The electrostatic potential $V(x,y)$ in free space in a region where the charge density ρ is zero is given by $V(x,y) = 4e^{2x} + f(x) - 3y^2$. Given that the x-component of the electric field, E_x, and V are zero at the origin, $f(x)$ is:

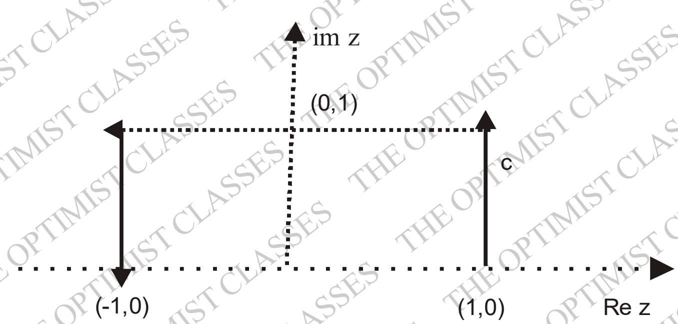
(a) $3x^2 - 4e^{2x} + 8x$ (b) $3x^2 - 4e^{2x} + 16x$ (c) $4e^{2x} - 8x$ (d) $3x^2 - 4e^{2x}$

24. Consider the transition of liquid water to steam as water boils at a temperature of 100°C under a pressure of 1 atmosphere. Which one of the following quantities does not change discontinuously at the transition?

(a) The Gibbs free energy (b) The interval energy

(c) The entropy (d) The specific volume

25. The value of the integral $\int_C z^2 e^z dz$, where C is an open contour in the complex Z-plane as shown in figure below:



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$ (c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

26. Which of the following matrices is an elecment of the group $SU(2)$?

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$ (c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

27. For constant uniform electric and magnetic fields $\vec{E}=\vec{E}_0$ and $\vec{B}=\vec{B}_0$, it is possible to choose a gauge such that the scalar potential and vector potential ϕ and vector potential $\vec{A}$ are given by

(a) $\phi=0$ and $\vec{A}=\frac{1}{2}\left(\vec{B}_0\times\vec{r}\right)$ (b) $\phi=\vec{E}_0.\vec{r}$ and $\vec{A}=\frac{1}{2}\left(\vec{B}_0\times\vec{r}\right)$

(c) $\phi=-\vec{E}_0.\vec{r}$ and $\vec{A}=0$ (d) $\phi=0$ and $\vec{A}=-\vec{E}_0 t$

28. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the components of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by

(a) $\frac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{a^2}$ (b) $\frac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{b^2}$ (c) $\frac{\left(\vec{a}.\vec{b}\right)\vec{b}}{b^2}$ (d) $\frac{\left(\vec{b}.\vec{a}\right)\vec{a}}{a^2}$

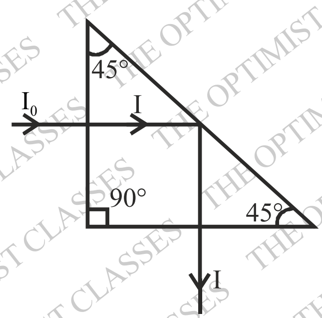
29. The wavefunction of a particle is given by $\psi=\left(\frac{1}{\sqrt{2}}\phi_0+i\phi_1\right)$, where ϕ_0 and ϕ_1 are the normalized eigenfunctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state ψ is

(a) $\frac{E_0}{2}+E_1$ (b) $\frac{E_0}{2}-E_1$ (c) $\frac{E_0-2E_1}{3}$ (d) $\frac{E_0+2E_1}{3}$

30. A particle is confined to the region $x\geq 0$ by a potential which increases linearly as $u(x)=u_0x$. The mean position of the particle at temperature T is:

(a) $\frac{k_BT}{u_0}$ (b) $\left(k_BT\right)^2/u_0$ (c) $\sqrt{\frac{k_BT}{u_0}}$ (d) u_0k_BT

31. Circularly polarized light with intensity I_0 is incident normally on a glass prism as shown in the figure. The index of refraction of glass is 1.5. The intensity I of light emerging from the prism is:



(a) I_0 (b) $0.92\ I_0$ (c) $2.3\ I_0$ (d) $0.88\ I_0$.

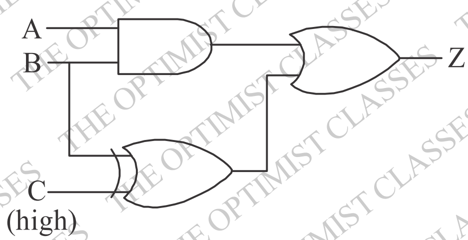
32. The acceleration due to gravity (g) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half that the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately:
(a) 1.1 (b) 1.3 (c) 2.3 (d) 5.2

33. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by $E(x,y,z,t) = E_0(\hat{x} + A\hat{z})\exp\left[ik_0 - \left\{ct + \left(x + \sqrt{3}z\right)\right\}\right]$, where c is the speed of light in vacuum E_0, A and k_0 are constants $\hat{x}$ and $\hat{z}$ are unit vectors along the x- and z-axes. The relative dielectric constant of the medium ε_r, and the constant A are

(a) $\varepsilon_r = 4$ and $A = -\frac{1}{\sqrt{3}}$ (b) $\varepsilon_r = 4$ and $A = +\frac{1}{\sqrt{3}}$

(c) $\varepsilon_r = 4$ and $A = \sqrt{3}$ (d) $\varepsilon_r = 4$ and $A = -\sqrt{3}$

34. Consider the digital circuit shown below in which the input C is always high (I).



The truth table for the circuit can be written as

A	B	Z
0	0	
0	1	
1	0	
1	1	

The entries in the Z column (vertically) are
(a) 1011 (b) 0100 (c) 1111 (d) 1010

35. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$ are given by $E_{n\ell m} \propto -1/n^2$, where n is the principal qunatum number, and the corresponding wave functions are given by $\psi_{n\ell m}$ where ℓ is the orbital angular momentum qunatum number and m is the magnetic qunatum number. The spin of the electron is not considered. Which of the following is a correct statement?

(a) There are exactly $(2\ell + 1)$ different wave functions $\psi_{n\ell m}$ for each $E_{n\ell m}$.

(b) There are $\ell(\ell + 1)$ different wave functions $\psi_{n\ell m}$ for each $E_{n\ell m}$.

(c) $E_{n\ell m}$ does not depend on ℓ and m for the Coulomb potential.

(d) There is a unique wave function $\psi_{n\ell m}$ and $E_{n\ell m}$.

36. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}.\vec{B}$ where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the 2×2 unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to

(a) $I\cos\frac{\omega t}{2}+\frac{i\vec{\sigma}.\vec{B}}{B}\sin\frac{\omega t}{2}$ (b) $I\cos\omega t+\frac{i\vec{\sigma}.\vec{B}}{B}\sin\omega t$

(c) $I\sin\omega t+\frac{i\vec{\sigma}.\vec{B}}{B}\cos\omega t$ (d) $I\sin 2\omega t+\frac{i\vec{\sigma}.\vec{B}}{B}\cos 2\omega t$

37. The Hamiltonian of a system with n degrees of freedom is given by

$H(q_1,.........,q_n;p_1,.........,p_n;t)$,

with an explicit dependence on the time t. Which of the following is correct?

(a) Different phase trajectories cannot intersect each other.

(b) H always represents the total energy of the system and is a constant of the motion.

(c) The equations $\dot{q}_i=\partial H/\partial p_i,\ \dot{p}_i=-\partial H/\partial q_i$ are not valid since H has explicit time dependence.

(d) Any initial volume element in phase space remains unchanged in magnitude under time evolution.

38. If the perturbation $H'=ax$, where a is a constant is added to the infintite square well potential

$$V(x)=\begin{cases}0 & \text{for} \quad 0\le x\le\pi\\ \infty & \text{otherwise}\end{cases}$$

The first order correction to ground state energy is:

(a) $\frac{a\pi}{2}$ (b) $a\pi$ (c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

39. Let $p_n(x)$ (where $n=0,1,2$ ....) be a polynomial of degree n with real coefficients, defined in the interval $2\le n\le 4$. If $\int_2^4 p_n(x)p_m(x)\,dx=\delta_{nm}$, then

(a) $p_0(x)=\frac{1}{\sqrt{2}}$ and $p_1(x)=\sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x)=\frac{1}{\sqrt{2}}$ and $p_1(x)=\sqrt{3}(3+x)$

(c) $p_0(x)=\frac{1}{2}$ and $p_1(x)=\sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x)=\frac{1}{\sqrt{2}}$ and $p_1(x)=\sqrt{\frac{3}{2}}(3-x)$

40. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V=\gamma T^3$ where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is:

(a) $4\gamma T^4$ (b) $2\gamma T^4$ (c) γT^4 (d) $\frac{\gamma T^4}{4}$

PART - C

41. Consider a system of N non-interaciting spins, each of which has classical magnetic moment of magnitude μ. The Hamiltonian of this system in an external magnetic field $\vec{H}$ is $H=-\sum_{i-1}^{N}\vec{\mu}_i.\vec{H}$, where $\vec{\mu}_i$ is the magnetic moment of the i[th] spin. The magnetization per spin at temperature T is:

(a) $\frac{\mu^2 H}{k_B T}$ (b) $\mu\left[\coth\left(\frac{\mu H}{k_B T}\right)-\frac{k_B T}{\mu H}\right]$

(c) $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$ (d) $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$

42. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$?

(a) $(3+x-iy)^7$ (b) $(1-x-iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

43. A particle in one dimension moves under the influence of a potential $V(x)=ax^6$, where a is a real constant. For large n the quantized energy level E_n depends on n as:

(a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$ (c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

44. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic field is given by $L = \frac{1}{2}m\vec{v}^2 + e\vec{A}.\vec{v} - e\phi$, where $\vec{A}$ and ϕ are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct?

(a) The canonically conjugate momentum of the particle is given by $\vec{p} = m\vec{v}$.

(b) The Hamiltonian of the particle is given by $H = \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{A}.\vec{p} + e\phi$

(c) L remains unchanged under a gauge transformation of the potentials.

(d) Under a gauge transformation of the potentials, L changes by the total times derivative of a function of $\vec{r}$ and t.

45. A static, spherically symmetric charge distribution is given by $\rho(r) = \frac{A}{r}e^{-kr}$ where A and k are positive constants. The electostatic potential corresponding to this charge distribution varies with r as

(a) $r\, e^{-kr}$ (b) $\frac{1}{r}e^{-kr}$ (c) $\frac{1}{r^2}e^{-kr}$ (d) $\frac{1}{r}\left(1-e^{-kr}\right)$

46. Consider two independently diffusion non-interacting particle in 3-dimensional space, both placed at the origin at time $t=0$. These particles have different diffusion constants D_1 and D_2. The quantity $\left\langle\left[\vec{R}_1(t)-\vec{R}_2(t)\right]^2\right\rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as:

(a) $6t(D_1+D_2)$ (b) $6t(D_1-D_2)$ (c) $6t\sqrt{D_1^2+D_2^2}$ (d) $6t\sqrt{D_1 D_2}$

47. A resistance is measured by passing current through it and measuring the resulting voltage drop. If the voltmeter and the ammeter have uncertainties of 3% and 4%, respectively, then

(A) The uncertainty in the value of resistance is:

(a) 7.0% (b) 3.5% (c) 5.0% (d) 12.0%

(B) The uncertainty in the computed value of the power dissipated in resistance is

(a) 7% (b) 5% (c) 11% (d) 9%

48. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given by I_1, I_2 and I_3 is rotating freely about a fixed point inside the body. The Euler equations for the components of its angular velocity $(\omega_1, \omega_2, \omega_3)$ are $\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3, \dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3, \dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_1\omega_2$

(A) The equilibrium points in $(\omega_1, \omega_2, \omega_3)$ space are

(a) (1,-1,0), (-1,0,1) and (0,-1,1) (b) (1,1,0), (1,0,1) and (0,1,1)
(c) (1,0,0), (0,1,0) and (0,0,1) (d) (1,1,1), (-1,-1,-1) and (0,0,0)

(B) The constants of motion are

(a) $\omega_1^2+\omega_2^2+\omega_3^2$ and $I_1\omega_1+I_2\omega_2+I_3\omega_3$ (b) $I_1\omega_1^2+I_2\omega_2^2+I_3\omega_3^2$ and $I_1^2\omega_1^2+I_2^2\omega_2^2+I_3^2\omega_3^2$

(c) $I_1\omega_1^2+I_2\omega_2^2+I_3\omega_3^2$ and $\omega_1+\omega_2+\omega_3$ (d) $\omega_1^2+\omega_2^2+\omega_3^2$ and $I_1^2\omega_1^2+I_2^2\omega_2^2+I_3^2\omega_3^2$

49. In a system consisting of two spin $-\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)}=\frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)}=\frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators, Here $\vec{\sigma}\left(\sigma_x,\sigma_y,\sigma_z\right)$ and $\sigma_x,\sigma_y,\sigma_z$ are ther three Pauli matrices.

(A) In the standard basis the matries for the operators $S_x^{(1)}S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ respectively,

(a) $\frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ (b) $\frac{\hbar^2}{4}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$

(c) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & -i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & -i & 0\\ 0 & i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}$ (d) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

(B) These two operators satisfy the relation

(a) $\left\{S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right\}=S_z^{(1)}S_z^{(2)}$ (b) $\left\{S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right\}=0$

(c) $\left\{S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right\}=i\,S_z^{(1)}S_z^{(2)}$ (d) $\left[S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right]=0$

50. Consider the matrix $M=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$

(A) The eigenvalues of *M* are

(a) 0,1,2 (b) 0,0,3 (c) 1,1,1 (d) -1,1,3

(B) The exponential of *M* simplifies to (*I* is the 3×3 identity matrix)

(a) $e^M=I+\left(\frac{e^3-1}{3}\right)M$ (b) $e^M=I+M+\frac{M^2}{2!}$

(c) $e^M=I+3^3M$ (d) $e^M=(e-1)M$

51. The radius of a $^{64}_{29}Cu$ nucleus is measured to be 4.8×10^{-13} cm.

(A) The radius of a $^{27}_{12}\text{Mg}$ nucleus can be estimated to be

(a) 2.86×10^{-13} cm (b) 5.2×10^{-13} cm (c) 3.6×10^{-13} cm (d) 8.6×10^{-13} cm

(B) The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number *A* in its ground state varies as

(a) $A^{4/3}$ (b) $A^{1/3}$ (c) $A^{-1/3}$ (d) $A^{-2/3}$

52. The character table of C_3, the group of symmetries of a an equilateral traingle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1C_1$	1	1	b
$3C_2$	1	a	c
$2C_3$	1	1	d

In the above denotes the three classes of C_{3v}, containing 1,3 and 2 elements respectively, $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducile representations $\Gamma^{(0)}$, $\Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v}.

(A) The entries a, b, c and d in this table are, respectively

(a) 2,1,-1,0 (b) -1,2,0,-1 (c) -1,1,0,-1 (d) -1,1,1,-1

(B) The reducible representation Γ of C_{3v} with character $\chi = (4,0,1)$ decomposes into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (c) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (d) $2\Gamma^{(2)}$

53. Light of wavelength 660nm and power of 1mW is incident on a semiconductor photodiode with an absorbing layer of thickness of (In 4) μm .

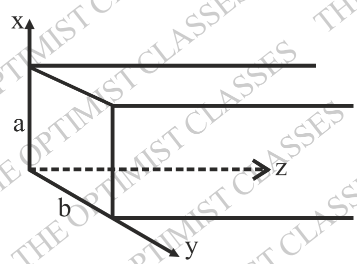
(A) If the absorption coefficient at this wavelength is 10^4 cm^{-1} and if 1% power is lost on reflection at the surface, the power absorbed will be

(a) 750 μm (b) 675 μm (c) 250 μm (d) 225 μm

(B) The generated photo-current for a quantum effeciency of unity will be

(a) 400 μA (b) 360 μA (c) 133 μA (d) 120 μA

54. The magnetic field of the TE_{11} mode of a rectangular waveguide of dimensions $a \times b$ as shown in the figure is given by $H_z = H_0 \cos(0.3\pi x)\cos(0.4\pi y)$, where *x* and *y* are in cm.



(A) The dimensions of the waveguide are

(a) a = 3.33 cm, b = 2.50 cm (b) a = 0.40 cm, b = 0.30 cm

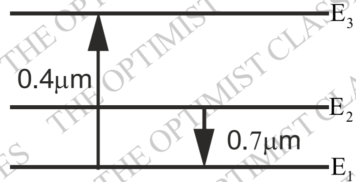
(c) a = 0.80 cm, b = 0.60 cm (d) a = 1.66 cm, b = 1.25 cm

(B) The entire range of frequencies *f* for which the TE_{11} mode will propagate is:

(a) 6.0 GHz < f < 7.5 GHz (b) 7.5 GHz < f < 9.0 GHz

(c) 7.5 GHz < f < 12.0 GHz (d) 7.5 GHz < f

55. Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with 1.6×10^{19} Chromium ions per cubic centimeter. All the atoms excited by the $0.4\,\mu$m radiation decay rapidly to level E_2 which has a lifetime $\tau = 3$ ms



(A) Assuming that there is no radiation of wavelength 0.7μ m present in the pumping cycle and that the pumping rate is R atoms per cm^3, the population density in the level N_2 builds up as:

(a) $N_2(t)=R\tau\left(e^{t/\tau}-1\right)$ (b) $N_2(t)=R\tau\left(1-e^{-t/\tau}\right)$

(c) $N_2(t)=\frac{Rt^2}{\tau}\left(1-e^{-t/\tau}\right)$ (d) $N_2(t)=Rt$

(B) The minimum pump power required (per cubic centimeter) to bring the system to transparency, i.e. zero gain, is

(a) 1.52 kW (b) 2.64 kW (c) 0.76 kW (d) 1.32 kW

56. A flux quantum (fluxoid) is approximately equal to 2×10^{-7} gauss-cm^2. A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor.

The strength of the field at this point is $\frac{2}{\pi}\times10^5$ gauss

(A) The penetration depth of this superconductor is

(a) 100Å (b) 10Å (c) 1000Å (d) 314Å

(B) The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi}\times10^5$ gauss. The correlation length of the superconductor is:

(a) 20Å (b) 200Å (c) 682Å (d) 2000Å

57. A beam of pions (π^+) is incident on a proton target, giving rise to the process $\pi^+p\to n+\pi^++\pi^+$

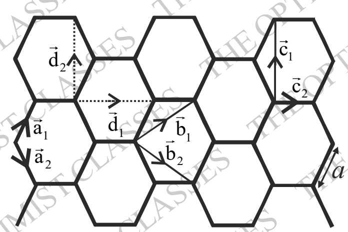
(A) Assuming that the decay proceeds through strong interactions, the total isospin I and its third component I_3 for the decay product, are

(a) $I=\frac{3}{2},I_3=\frac{3}{2}$ (b) $I=\frac{5}{2},I_3=\frac{5}{2}$ (c) $I=\frac{5}{2},I_3=\frac{3}{2}$ (d) $I=\frac{1}{2},I_3=\frac{1}{2}$

(B) Using isospin symmetry, the cross-section for the above process can be related to that of the process

(a) $\pi^-n\to p\pi^-\pi^-$ (b) $\pi^-\overline{p}\to\overline{n}\pi^-\pi^-$ (c) $\pi^+n\to p\pi^+\pi^-$ (d) $\pi^+\overline{p}\to n\pi^+\pi^-$

58. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing a, as shown below. The carbon atoms occupy the vertices.



(A) The Wigner-Seitz cell has an area of

(a) $2a^2$ (b) $\frac{\sqrt{3}}{2}a^2$ (c) $6\sqrt{3}a^2$ (d) $\frac{3\sqrt{3}}{2}a^2$

(B) The Bravais lattice for this array is a

(a) Rectangular lattice with basis vectors $\vec{d}_1$ and $\vec{d}_2$

(b) Rectangular lattice with basis vectors $\vec{c}_1$ and $\vec{c}_2$

(c) Hexagonal lattice with basis vectors $\vec{a}_1$ and $\vec{a}_2$

(d) Hexagonal lattice with basis vectors $\vec{b}_1$ and $\vec{b}_2$

59. Consider the decay process $\tau^- \to \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of τ^-, π^- and ν_τ are M_τ, M_π and zero respectively.

(A) The energy of π^- is:

(a) $\frac{\left(M_\tau^2 - M_\pi^2\right)c^2}{2M_\tau}$ (b) $\frac{\left(M_\tau^2 + M_\pi^2\right)c^2}{2M_\tau}$ (c) $\left(M_\tau - M_\pi\right)c^2$ (d) $\sqrt{M_\tau M_\pi}c^2$

(B) The velocity is π^- is

(a) $\frac{\left(M_\tau^2 - M_\pi^2\right)c}{M_\tau^2 + M_\pi^2}$ (b) $\frac{\left(M_\tau^2 - M_\pi^2\right)c}{M_\tau^2 - M_\pi^2}$ (c) $\frac{M_\pi c}{M_\tau}$ (d) $\frac{M_\tau c}{M_\pi}$

60. A narrow beam of X-rays with wavelength 1.5 Å is reflected from an ionic crystal with an fcc lattice structure with a density of 3.32 g cm^{-3}. The molecular weight is 108 AMU $\left(1\,\text{AMU} = 1.66\times10^{-24}\,\text{g}\right)$.

(A) The lattice constant is:

(a) 6.00 Å (b) 4.56 Å (c) 4.00 Å (d) 2.56 Å

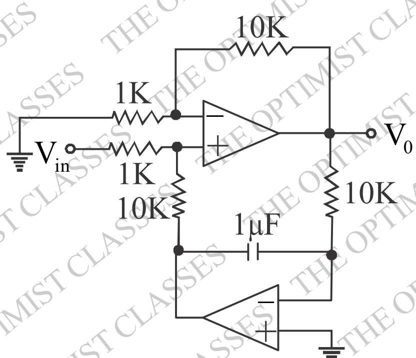
(B) The sine of the angle corresponding to (111) reflection is:

(a) $\sqrt{3}/4$ (b) $\sqrt{3}/8$ (c) 1/4 (d) 1/8

61. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately

(a) 0.68 (b) 0.48 (c) 0.28 (d) 0.91

62. A time varying signal V_{in} is fed to an op-amp circuit with output signal V_0 as shown in the figure below. The circuit implements a



(a) High pass filter with cutoff frequency 16 Hz.

(b) High pass filter with cutoff frequency 100 Hz.
(c) Low pass filter with cut off frequency 16 Hz.
(d) Low pass filter with cut off frequency 100 Hz.

63. The Hamitonian of a particle of unit mass moving in the *xy*-plane is given to be:
$H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$ in suitable units. The initial values are given to be $\left(x(0), y(0)\right) = (1,1)$ and $\left(p_x(0), p_y(0)\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$. During the motion, the curves traced out by the particles in the *xy*-plane and the $p_x p_y$-plane are
(a) Both strainght lines (b) A straight line and a hyperbola respectively
(c) A hyperbola an ellipse, respectively (d) Both hyperbolas

64. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s > 0$. The range of s for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is:
(a) $1 < s < 3$ (b) $0 < s < 2$ (c) $0 < s < 3$ (d) $0 < s < \infty$

65. Two gravitating bodies *A* and *B* with masses m_A and m_B respectively, are moving in circular orbit. Assume that $m_B \gg m_A$ and let the radius of the orbit of body *A* be R_A. If the body *A* is losing mass adiabatically, its orbital radius R_A is proportional to
(a) $1/m_A$ (b) $1/m_A^2$ (c) m_A (d) m_A^2

ANSWER KEY

21. (b)	22. (c)	23. (d)	24. (a)	25. (c)	26. (b)	27. (b)
28. (a)	29. (d)	30. (a)	31. (c)	32. (c)	33. (a)	34. (d)
35. (c)	36. (b)	37. (d)	38. (a)	39. (d)	40. (d)	41. (b)
42. (b)	43. (d)	44. (d)	45. (b)	46. (a)	47. (b)	48. (c,b)
49. (c,d)	50. (b,a)	51. (c,c)	52. (b,c)	53. (c,c)	54. (a,d)	55. (b,d)
56. (a,a)	57. (c,b)	58. (d,c)	59. (b,a)	60. (a,b)	61. (a)	62. (a)
63. (d)	64. (c)	65. (b)				