THE OPTIMIST CLASSES
IIT-JAM TOPPERS

8 AIR

MANOJ KUMAR SINGH

9 AIR

ABHAY

12 AIR

PAWAN

15 AIR

SATYAM

21 AIR

SOUMIL GIRISH SAHU

29 AIR

BHOOMIJA

31 AIR

AKSHIT AGGARWAL

32 AIR

SHIKHAR CHAMOLI

33 AIR

RAVI SINGH ADHIKARI

44 AIR

GAURAV JHA

56 AIR

SWAPNIL JOSHI

65 AIR

LOKESH BHATT

75 AIR

GOPESH VISHVAKARMA

77 AIR

VAIBHAV

80 AIR

SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022

ANNU
DL01000308

ALANKAR
UP15000162

SAHIL RANA
HR09000108

JAYESTHI
RJ11000161

DASRATH
RJ06000682

VIVEK
UK01000439

UZAIR AHMED
UP02000246

SURYA PRATAP SINGH
RJ06000232

HIMANSHU
UP10000095

CHANDAN
RJ09000159

SAIKHOM JOHNSON
MN01000196

AJAY SAINI
RJ06001744

VIKAS YADAV
RJ06001102

JYOTSNA KOHLI
UK02000262

SHYAM SUNDAR
RJ06000615

CSIR-UGC-NET/JRF-DECEMBER-2012

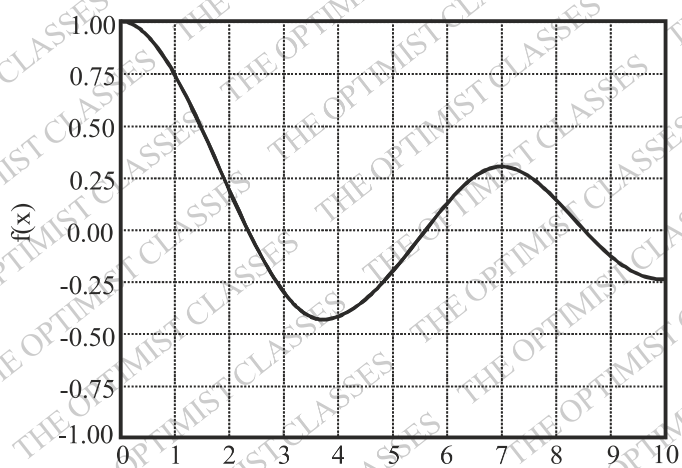
PREVIOUS YEAR QUESTION

PHYSICAL SCIENCES

PART-B

21. A 2×2 matrix A has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = 1$ is:
(a) 20 (b) 30 (c) 60 (d) 120

22. The graph of the function $f(x)$ as shown below is best described by



(a) The bessel function $J_0(x)$ (b) $\cos x$
(c) $e^{-x}\cos x$ (d) $\frac{1}{x}\cos x$

23. In a series of five cricket matches, one of the captains calls ''Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is
(a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

24. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}},\frac{b}{\sqrt{3}},\frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

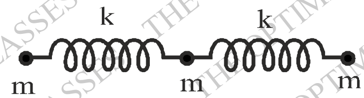
(a) $\frac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (b) $\frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (c) $\frac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

25. A solid cylinder of height H, radius R and density ρ, floats vertically on the surface of a liquid of density ρ_0. The cylinder will be set into oscillatory motion when a small instantaneous downward

force is applied. The frequency of oscillation is

(a) $\frac{\rho g}{\rho_0 H}$ (b) $\frac{\rho}{\rho_0}\sqrt{\frac{g}{H}}$ (c) $\sqrt{\frac{\rho g}{\rho_0 H}}$ (d) $\sqrt{\frac{\rho_0 g}{\rho H}}$

26. Three particles of equal mass 'm' are connected by two identical massless springs of stiffness constant 'k' as shown in the figure.



If x_1, x_2 and x_3 denote the displacements of the masses from their respective equilibrium positions, the potential energy of the system is:

(a) $\frac{1}{2}k\left(x_1^2+x_2^2+x_3^2\right)$ (b) $\frac{1}{2}k\left[x_1^2+x_2^2+x_3^2-x_2\left(x_1+x_3\right)\right]$

(c) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2+2x_2\left(x_1+x_3\right)\right]$ (d) $\frac{1}{2}k\left[x_1^2+2x_2^2+x_3^2-2x_2\left(x_1+x_3\right)\right]$

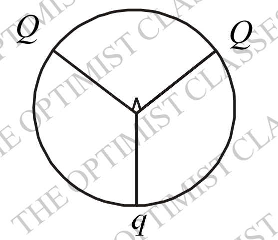
27. Let v, p and E denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass 'm'. Then

(a) $\frac{dE}{dp}$ = constant (b) $p = mv$ (c) $v = \frac{cp}{\sqrt{p^2+m^2c^2}}$ (d) $E = mc^2$

28. A binary star system consists of two stars S_1 and S_2, with masses m and $2m$ respectively separaterd by a distannce 'r'. If both S_1 and S_2 individually follow circular orbits around the centre of the mass with intantaneous speeds v_1 and v_2 respectively, the ratio of speeds v_1/v_2 is:

(a) $\sqrt{2}$ (b) 1 (c) 1/2 (d) 2

29. Three charges are located on the circumference of a circle of radius 'R' as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge 'q' is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?



(a) $q/\sqrt{2}$ (b) $\sqrt{2}q$ (c) 2q (d) 4q

30. Considser a hollow charged shell of inner radius 'a' and outer radius 'b'. The volume charge density is $\rho(r) = \frac{k}{r^2}$ (where k is a constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is :

(a) $\frac{k(b-a)}{\varepsilon_0 r^2}$ $for\ r > a$ (b) $\frac{k(b-a)}{\varepsilon_0 r^2}$ $for\ a < r < b\ and\ \frac{kb}{\varepsilon_0 r^2}$ $for\ r > b$

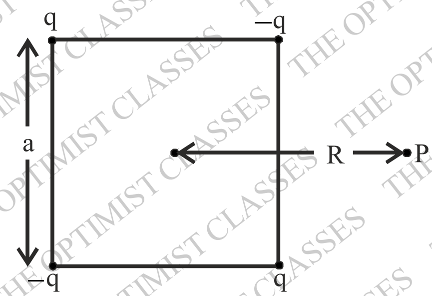
(c) $\frac{k(r-a)}{\varepsilon_0 r^2}$ *for* $a<r<b$ *and* $\frac{k(b-a)}{\varepsilon_0 r^2}$ *for* $r>b$

(d) $\frac{k(r-a)}{\varepsilon_0 a^2}$ *for* $a<r<b$ *and* $\frac{k(b-a)}{\varepsilon_0 a^2}$ *for* $r>b$

31. Consider the interfernce of two coherent electromagnetic waves whose electric field vectors are given by $\vec{E}_1=\hat{i}E_0\cos\omega t$ and $\vec{E}_2=\hat{j}E_0\cos(\omega t+\varphi)$ where φ is the phase difference. The intensity of the resulting wave is given by $\frac{\varepsilon_0}{2}\langle E^2\rangle$, where $\langle E^2\rangle$ is the time average of E^2. The total intensity is

(a) 0 (b) $\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2\sin^2\varphi$ (d) $\varepsilon_0 E_0^2\cos^2\varphi$

32. Four charges (two $+q$ and two $-q$) are kept fixed at the four vertices of a square of side 'a' as shown



At the point P which is at a distance R from the centre ($R>>a$), the potential s proportional to

(a) $\frac{1}{R}$ (b) $\frac{1}{R^2}$ (c) $\frac{1}{R^3}$ (d) $\frac{1}{R^4}$

33. A point charge 'q' of mass 'm' is kept at a distance 'd' below a grounded inifinite conducting sheet which lies in the xy-plane. What is the value of 'd' for which the charge remains stationary?

(a) $q/4\sqrt{mg\pi\varepsilon_0}$ (b) $q/\sqrt{mg\pi\varepsilon_0}$

(c) There is no finite value of 'd' (d) $\sqrt{mg\pi\varepsilon_0}/q$

34. The wave function of a state of the hydrogen atom is given by

$$\psi=\psi_{200}+2\psi_{211}+3\psi_{210}+\sqrt{2}\psi_{21-1}$$

where ψ_{nlm} denotes the normalized eigen function of the state with qunatum numbers n, l and m in the usual notation. The expectation value of L_Z in the state ψ is:

(a) $\frac{15\hbar}{16}$ (b) $\frac{11\hbar}{16}$ (c) $\frac{3\hbar}{8}$ (d) $\frac{\hbar}{8}$

35. The energy eigenvalues of a particle in the potential $V(x)=\frac{1}{2}m\omega^2x^2-ax$ are

(a) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-\frac{a^2}{2m\omega^2}$ (b) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega+\frac{a^2}{2m\omega^2}$

(c) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-\frac{a^2}{m\omega^2}$ (d) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$

36. If a particle is represented by the normalized wave function

$$\psi(x)=\begin{cases}\dfrac{\sqrt{15}\left(a^2-x^2\right)}{4a^{5/2}} & \text{for } -a<x<a\\ 0 & \text{otherwise}\end{cases}$$

the uncertainty Δp in its momentum is

(a) $\frac{2\hbar}{5a}$ (b) $\frac{5\hbar}{2a}$ (c) $\frac{\sqrt{10}\hbar}{a}$ (d) $\frac{\sqrt{5}\hbar}{\sqrt{2}a}$

37. Given the usual canonical commutation relations, the commutator $[A,B]$ of $A=i\left(xp_y-yp_x\right)$ and $B=\left(yp_z+zp_y\right)$ is:

(a) $\hbar\left(xp_z-p_x z\right)$ (b) $-\hbar\left(xp_z-p_x z\right)$ (c) $\hbar\left(xp_z+p_x z\right)$ (d) $-\hbar\left(xp_z+p_x z\right)$

38. The entropy of a system, S, is related to the accessible phase space volume Γ by $S=k_B \ell n\Gamma(E,N,V)$ where E, N and V are the energy, number of particles and volume respectively. Form this one can conclude that Γ

(a) does not change during evolution to equilibrium
(b) Oscillates during evolution to equilibrium
(c) Is a maximum in equilibrium
(d) Is a minimum in equilibrium

39. Let ΔW be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about ΔW is correct?

(a) ΔW is a perfect differential if the process is isothermal
(b) ΔW is a perfect differential if the process is adiabatic
(c) ΔW is always a perfect differential.
(d) ΔW cannot be a perfect differential.

40. Consider a system of three spins S_1, S_2 and S_3 each of which can take values +1 and -1. The energy of the system is given by $E=-J\left[S_1S_2+S_2S_3+S_3S_1\right]$, where J is a positive constant. The minimum energy and the corresponding number of spin configurations are, respectively,

(a) J and 1 (b) -3J and 1 (c) -3J and 2 (d) -6J and 2

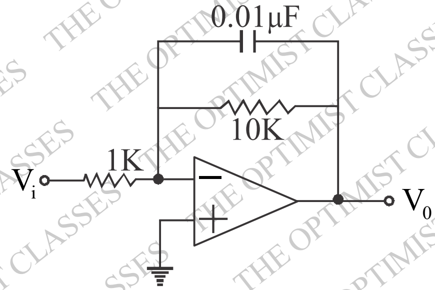
41. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is

(a) $14\pi^2\hbar^2/mL^2$ (b) $91\pi^2\hbar^2/mL^2$ (c) $7\pi^2\hbar^2/mL^2$ (d) $3\pi^2\hbar^2/mL^2$

42. A live music broadcast consists of a radio-wave of frequency 7MHz, amplitude-modulated by a microphone output consisting of signals with a maximum frequency of 10 KHz. The spectrum of modulated output will be zero outside the frequency band

(a) 7.00 MHz to 7.01 MHz (b) 6.99 MHz to 7.01 MHz
(c) 6.99 MHz to 7.00 MHz (d) 6.995 MHz to 7.005 MHz

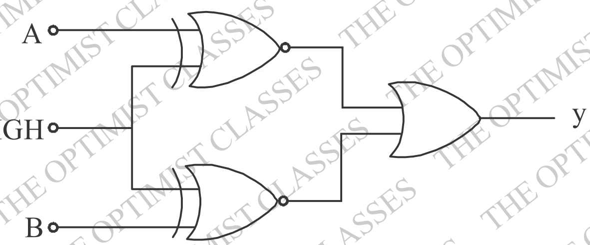
43. In the op-amp circuit shown in the figure, V_i is a sinusoidal input signal of frequency 10 Hz and V_0 is the output signal.



The magnitude of the gain and the phase shift, respectively, are close to the values

(a) $5\sqrt{2}$ and $\frac{\pi}{2}$ (b) $5\sqrt{2}$ and $\frac{-\pi}{2}$ (c) 10 and zero (d) 10 and π

44. The logic circuit shown in the figure below

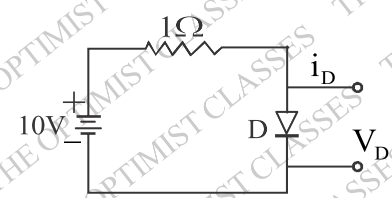


implements the Boolean expression

(a) $y=\overline{A.B}$ (b) $y=\bar{A}.\bar{B}$ (c) $y=A.B$ (d) $y=A+B$

45. A diode D as shown in the circuit as an $i-v$ relation which can be approximated by

$$i_D=\begin{cases} v_D^2+2v_D, & \text{for } v_D>0 \\ 0, & \text{for } v_D\le 0 \end{cases}$$



The value of v_D in the circuit is:

(a) $\left(-1+\sqrt{11}\right)$V (b) 8V (c) 5V (d) 2V

PART-C

46. The Taylor expansion of the function $\ell n(\cosh x)$, where 'x' is real, about the point x = 0 starts with the following terms:

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+.........$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+.........$

(c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+.........$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+.........$

47. Given a 2×2 unitary matrix U satisfying $U^\dagger U=UU^\dagger=1$ with det $U=e^{i\varphi}$, one can construct a unitary matrix $V\left(V^\dagger V=VV^\dagger=1\right)$ with det V = 1 from it by

(a) Multiplying U be $e^{-i\varphi/2}$ (b) Multiplying any single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column of U by $e^{-i\varphi/2}$ (d) Multiplying U by $e^{-i\varphi}$

48. The value of the integral $\int_C \frac{z^3 dz}{z^2-5z+6}$, where C is a closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction, is:

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

49. A function f(x) obeys the differential equation and satisfies the conditions $\frac{d^2f}{dx^2}-(3-2i)f=0$ and satisfies the conditions $f(0)=1$ and $f(x)\to 0$ as $x\to\infty$. The value of $f(\pi)$ is:

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi i}$

50. A planet of mass 'm' moves in the gravitational field of Sun (mass M). If the semi-major and semi-minor axes of the orbit are 'a' and 'b' respectively, the angular momentum of the planet is:

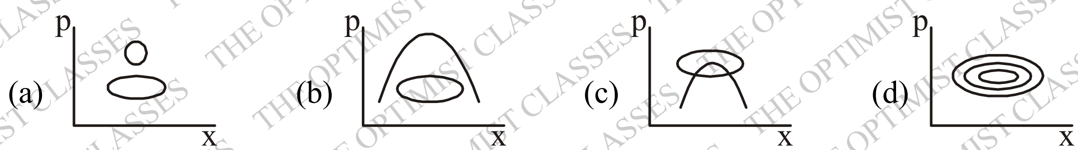
(a) $\sqrt{2GMm^2(a+b)}$ (b) $\sqrt{2GMm^2(a-b)}$

(c) $\sqrt{\frac{2GMm^2ab}{(a-b)}}$ (d) $\sqrt{\frac{2GMm^2ab}{(a+b)}}$

51. TheHamiltonian of a simple pendulum consisting of a mass 'm' attached to a massless string of length l is $H=\frac{p_\theta^2}{2m\ell^2}+mg\ell(1-\cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is:

(a) $-\frac{2g}{\ell}p_\theta\sin\theta$ (b) $-\frac{g}{\ell}p_\theta\sin 2\theta$ (c) $\frac{g}{\ell}p_\theta\cos\theta$ (d) $\ell p_\theta^2\cos\theta$

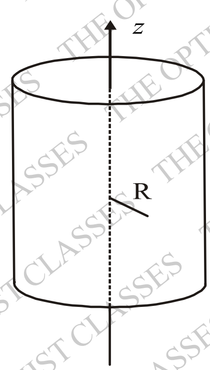
52. Which of the following set of phase -space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian) ?



53. Two bodies of equal mass 'm' are connected by a massless rigid rod of length 'l' lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is

(a) $m\ell^2\omega/4$ (b) $m\ell^2\omega/2$ (c) $m\ell^2\omega$ (d) $2m\ell^2\omega$

54. An infinite solenoid with its axis of symmetry along the z- direction carries a steady current I.



The vector potential $\vec{A}$ at a distance R from the axis.

(a) Is constant inside and varies as R outside the solenoid.
(b) Varies as R inside and is constant outside the solenoid.
(c) Varies as 1/R inside and R outside the solenoid.
(d) Varies as R inside and as 1/R outside the solenoid.

55. Consider an infinite conducting sheet in the xy-plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at (x,y,z) is given by

$$\vec{A} = \frac{\mu_0 K}{4c}(ct-z)^2\hat{i}$$

The magnetic field $\vec{B}$ is:

(a) $\frac{\mu_0 K t}{2}\hat{j}$ (b) $-\frac{\mu_0 K z}{2c}\hat{j}$ (c) $-\frac{\mu_0 K}{2c}(ct-z)\hat{i}$ (d) $-\frac{\mu_0 K}{2c}(ct-z)\hat{j}$

56. When a charged particle emits electromagnetic radiation, the electric field $\vec{E}$ and the Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ at a large distance 'r' from the emitter vary as $\frac{1}{r^n}$ and $\frac{1}{r^m}$ respectively. Which of the following choices for 'n' and 'm' are correct?

(a) n = 1 and m = 1 (b) n = 2 and m = 2 (c) n = 1 and m = 2 (d) n = 2 and m = 4

57. The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential $V(x)$ are -4 and -1, respectively, (in units in which $\hbar = 1$). If the correspoding wavefunctions are related by $\psi_1(x) = \psi_0(x)\sinh x$, then the ground state eigenfunction is

(a) $\psi_0(x) = \sqrt{\operatorname{sech} x}$ (b) $\psi_0(x) = \operatorname{sech} x$

(c) $\psi_0(x) = \operatorname{sech}^2 x$ (d) $\psi_0(x) = \operatorname{sech}^3 x$

58. The perturbation

$$H' = \begin{cases} b(a-x) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

acts on a particle of mass 'm' confined in an infinite square well potential

$$V(x) = \begin{cases} 0 & -a < x < a \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to the ground state energy of the particle is

(a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$ (c) 2ba (d) ba

59. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is :

(a) $\Delta x = \sqrt{\hbar / 2m\omega}$ (b) $\Delta x = \sqrt{\hbar / m\omega}$ (c) $\Delta x = \sqrt{2\hbar / m\omega}$ (d) $\Delta x = \sqrt{\hbar / 4m\omega}$

60. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

if vibrational principle is used to estimate it with the trial wavefunction $\psi(x) = Ae^{-bx^2}$ with b as the variational parameter?

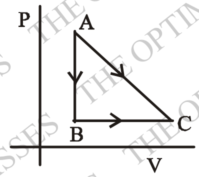
$$\left[\text{Hint}: \int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-\frac{1}{2}} \Gamma\left(n + \frac{1}{2}\right)\right]$$

(a) $-m\alpha^2 / 2\hbar^2$ (b) $-2m\alpha^2 / \pi\hbar^2$ (c) $-m\alpha^2 / \pi\hbar^2$ (d) $-m\alpha^2 / \pi\hbar^2$

61. The free energy difference between the superconducting and the normal states of a material is given by $\Delta F = F_S - F_N = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$, where ψ is an order parameter and α and β are constants such that $\alpha > 0$ in the normal and $\alpha < 0$ in the superconducting state, while $\beta > 0$ always. The minimum value of ΔF in the superconducting state is

(a) $-\alpha^2 / \beta$ (b) $-\alpha^2 / 2\beta$ (c) $-3\alpha^2 / 2\beta$ (d) $-5\alpha^2 / 2\beta$

62. A given quantity of gas is taken from the state $A \to C$ reversilbly, by two paths, $A \to C$ directly and $A \to B \to C$ as shown in the figure below:



During the $A \to C$ the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process $A \to B \to C$ the work done by the 30 J, the heat absored is :

(a) 20 J (b) 80 J (c) 220 J (d) 280 J

63. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2J. In a configuration with *r* spin flips, the energy of the system is E = -NJ + 2rJ and the number of configuration is $^{N}C_r$: r varies from 0 to N. The partition function is

(a) $\left(\frac{J}{k_B T}\right)^N$ (b) $e^{-NJ/k_B T}$ (c) $\left(\sinh\frac{J}{k_B T}\right)^N$ (d) $\left(\cosh\frac{J}{k_B T}\right)^N$

64. A magnetic field sensor based on the Hall effect is to be fabricated by implanting. As into a Si film of thickness 1 μm. The specifications require a magnetic field sensitivity of 500 m/VTesla at an excitation current of 1 mA. The implanation dose is to be adjusted such that the average carrier density, after activation, is

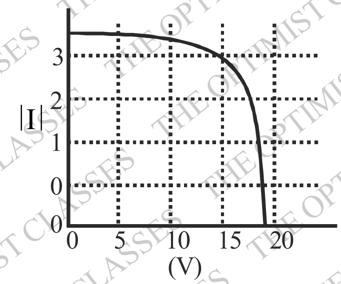
(a) $1.25\times10^{26}\ m^{-3}$ (b) $1.25\times10^{22}\ m^{-3}$ (c) $4.1\times10^{21}\ m^{-3}$ (d) $4.1\times10^{20}\ m^{-3}$

65. Band-pass and band-reject filters can be implemented by combining a low pass and a high pass filter in series and in parallel, respectively. If the cut-off frequencies of the low pass and high pass filters are ω_0^{LP} and ω_0^{HP}, respectively, the condition required to implement the band-pass and band-reject filters are respectively.

(a) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$ (b) $\omega_0^{HP} < \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

(c) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} < \omega_0^{LP}$ (d) $\omega_0^{HP} > \omega_0^{LP}$ and $\omega_0^{HP} > \omega_0^{LP}$

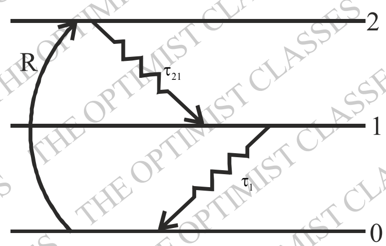
66. The output characteristics of a solar panel at a certain level of irradiance is shown in the figure below.



If the solar cell is to power a load of 5Ω, the power drawn by the load is:

(a) 97 W (b) 73 W (c) 50 W (d) 45 W

67. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is 10^{20} atoms cm^{-3} s^{-1} and the decay routes are as shown with $\tau_{21} = 20$ ns and $\tau_1 = 1\mu s$, the equilibrium populations of states 2 and 1 are, respectively,

(a) 10^{14} cm^{-3} and 2×10^{12} cm^{-3} (b) 2×10^{12} cm^{-3} and 10^{14} cm^{-3}

(c) 2×10^{12} cm^{-3} and 2×10^{6} cm^{-3} (d) zero and 10^{20} cm^{-3}.

68. Consider a hydrogen atom undergoing a $2P \rightarrow 1S$ transiton. The lifetime t_{sp} of the 2P state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium $n_0 = 1$, the ratio of the Einstein coefficients for stimulated emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

(a) 0.683×10^{12} $m^3J^{-1}s^{-1}$ (b) 0.146×10^{-12} $J\,s\,m^{-3}$

(c) 6.83×10^{12} $m^3J^{-1}s^{-1}$ (d) 1.463×10^{-12} $J\,s\,m^{-3}$

69. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance d = 20 cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes δv and the width Δv_p of each mode of the laser cavity are:

(a) $\delta v = 75\,kHz, \Delta v_p = 24\,kHz$ (b) $\delta v = 100\,kHz, \Delta v_p = 100\,kHz$

(c) $\delta v = 750\,MHz, \Delta v_p = 2.4\,MHz$ (d) $\delta v = 2.4\,MHz, \Delta v_p = 750\,MHz$

70. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to

(a) $\hbar\omega/2$ (b) $\hbar\omega$ (c) $3\hbar\omega/2$ (d) 0

71. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta\left(\cos k_x a + \cos k_y a + \cos k_z a\right)$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillioun zone is

(a) $\frac{2\hbar^2}{5\beta a^2}$ (b) $\frac{4\hbar^2}{5\beta a^2}$ (c) $\frac{\hbar^2}{2\beta a^2}$ (d) $\frac{\hbar^2}{3\beta a^2}$

72. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3, is

(a) $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ (b) $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$ (c) $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ (d) $\frac{1}{a}$

73. The muon has mass 105 MeV/c^2 and mean lifetime 2.2 μs in its rest frame. The mean distance traversed by muon of energy 315 MeV/c^2 before decaying is approximately

(a) 3×10^5 km (b) 2.2 cm (c) 6.6 μm (d) 1.98 km

74. Consider the following particles: the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of decreasing lifetime, the correct arrangement is an follows:

(a) π^0, n, p, Δ^+ (b) p, n, Δ^+, π^0 (c) p, n, π^0, Δ^+ (d) Δ^+, n, π^0, p

75. The single particle energy difference between the p-orbitals $\left(\text{i.e } p_{3/2} \text{ and } p_{1/2}\right)$ of the nucleus ${}^{114}_{50}\text{Sn}$ is 3 MeV. The energy difference between the states in its $1f$ orbital is

(a) -7 MeV (b) 7 MeV (c) 5 MeV (d) -5 MeV

ANSWER KEY

21. (c)	22. (a)	23. (d)	24. (a)	25. (d)	26. (d)	27. (c)
28. (d)	29. (a)	30. (c)	31. (b)	32. (c)	33. (a)	34. (d)
35. (a)	36. (d)	37. (c)	38. (c)	39. (b)	40. (c)	41. (a)
42. (b)	43. (d)	44. (a)	45. (d)	46. (b)	47. (a)	48. (a)
49. (c)	50. (*)	51. (a)	52. (c)	53. (b)	54. (d)	55. (d)
56. (c)	57. (c)	58. (d)	59. (a)	60. (c)	61. (b)	62. (b)
63. (d)	64. (b)	65. (b)	66. (d)	67. (b)	68. (a)	69. (c)
70. (c)	71. (d)	72. (a)	73. (d)	74. (c)	75. (b)	