# THE OPTIMIST CLASSES
# IIT-JAM TOPPERS

8 AIR

MANOJ KUMAR SINGH

9 AIR

ABHAY

12 AIR

PAWAN

15 AIR

SATYAM

21 AIR

SOUMIL GIRISH SAHU

29 AIR

BHOOMIJA

31 AIR

AKSHIT AGGARWAL

32 AIR

SHIKHAR CHAMOLI

33 AIR

RAVI SINGH ADHIKARI

44 AIR

GAURAV JHA

56 AIR

SWAPNIL JOSHI

65 AIR

LOKESH BHATT

75 AIR

GOPESH VISHVAKARMA

77 AIR

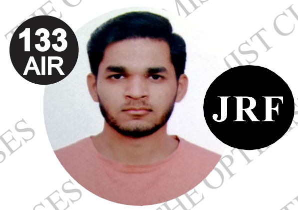
VAIBHAV

80 AIR

SHASWAT CHAMOLI

# CSIR-NET-JRF RESULTS 2022


**ANNU**
**DL01000308**




**ALANKAR**
**UP15000162**


**SAHIL RANA**
**HR09000108**


**JAYESTHI**
**RJ11000161**


**DASRATH**
**RJ06000682**


**VIVEK**
**UK01000439**


**UZAIR AHMED**
**UP02000246**


**SURYA PRATAP SINGH**
**RJ06000232**


**HIMANSHU**
**UP10000095**


**CHANDAN**
**RJ09000159**


**SAIKHOM JOHNSON**
**MN01000196**


**AJAY SAINI**
**RJ06001744**


**VIKAS YADAV**
**RJ06001102**


**JYOTSNA KOHLI**
**UK02000262**


**SHYAM SUNDAR**
**RJ06000615**

# THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

**CONTACT: 9871044043**

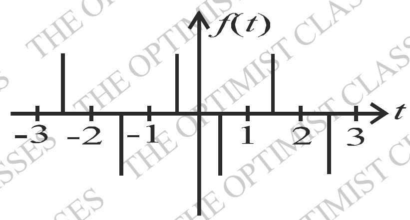
## CSIR-UGC-NET/JRF-JUNE-2015

## PREVIOUS YEAR QUESTION

### PHYSICAL SCIENCES

**Part - B**

21. Consider the periodic function $f(t)$ with time period $T$ as shown in the fugure below:




The spikes, are located at $t=\frac{1}{2}(2n-1)$, where $n=0,\pm1,\pm2,........,$ are Dirac delta functions of strength $\pm1$. The amplitudes $a_n$ in the fourier expansion $f(t)=\sum_{n=-\infty}^{\infty}a_n e^{2\pi \text{int}/T}$ are given by

(a) $(-1)^n$ (b) $\frac{1}{n\pi}\sin\frac{n\pi}{2}$ (c) $i\sin\frac{n\pi}{2}$ (d) $n\pi$

22. A particle moves in two dimensions on the ellipse $x^2+4y^2=8$. At a particular instant it is at the point $(x, y)=(2,1)$ and the $x$-component of its velocity is 6 (in suitable units). Then the $y$-component of its velocity is

(a) -3 (b) -2 (c) 1 (d) 4

23. Consider the differential equation $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=0$. If $x=0$ at $t=0$ and $x=1$ at $t=1$, the value of $x$ at $t=2$ is

(a) $e^2+1$ (b) $e^2+e$ (c) $e+2$ (d) $2e$

24. The value of integral $\int_{-\infty}^{\infty}\frac{dx}{1+x^4}$ is

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\sqrt{2}\pi$ (d) $2\pi$

25. The Laplace transform of $6t^3 + 3\sin 4t$ is

(a) $\frac{36}{s^4}+\frac{12}{s^2+16}$ (b) $\frac{36}{s^4}+\frac{12}{s^2-16}$ (c) $\frac{18}{s^4}+\frac{12}{s^2-16}$ (d) $\frac{36}{s^3}+\frac{12}{s^2+16}$

26. If the Lagrangian of a dynamical system in two dimensions is $L=\frac{1}{2}m\dot{x}^2+m\dot{x}\dot{y}$, then its Hamiltonian is

(a) $H=\frac{1}{m}p_x p_y+\frac{1}{2m}p_y^2$ (b) $H=\frac{1}{m}p_x p_y+\frac{1}{2m}p_x^2$

(c) $H=\frac{1}{m}p_x p_y-\frac{1}{2m}p_y^2$ (d) $H=\frac{1}{m}p_x p_y-\frac{1}{2m}p_x^2$

27. A particle of mass *m* moves in the one-dimensional potential $V(x)=\frac{\alpha}{3}x^3+\frac{\beta}{4}x^4$ where $\alpha,\beta>0$. One of the equilibrium points is $x=0$. The angular frequency of small oscillations about the other equilibrium point is

(a) $\frac{2\alpha}{\sqrt{3m\beta}}$ (b) $\frac{\alpha}{\sqrt{m\beta}}$ (c) $\frac{\alpha}{\sqrt{12m\beta}}$ (d) $\frac{\alpha}{\sqrt{24m\beta}}$

28. A particle of unit mass moves in the *xy*-plane in such a way that $\dot{x}(t)=y(t)$ and $\dot{y}(t)=x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential

(a) $\frac{1}{2}(x^2+y^2)$ (b) $\frac{1}{2}(x^2-y^2)$ (c) $x+y$ (d) $x - y$

29. Consider three inertial frames of reference A, B, and C. The frame B moves with a velocity *c*/2 with respect to A and C moves with a velocity *c*/10 with respect to B in the same direction. The velocity of C as measured in A is

(a) $\frac{3c}{7}$ (b) $\frac{4c}{7}$ (c) $\frac{c}{7}$ (d) $\frac{\sqrt{3}c}{7}$

30. A plane electromagnetic wave is travelling along the positive z-direction. The maximum electric field along the *x*-direction is 10 V/m. The approximate maximum values of the power per unit area and the magnetic induction B, respectively, are

(a) $3.3\times10^{-7}$ watts/m$^2$ and 10 tesla (b) $3.3\times10^{-7}$ watts/m$^2$ and $3.3\times10^{-8}$ tesla

(c) 0.265 watts/m$^2$ and 10 tesla (d) 0.265 watts/m$^2$ and $3.3\times10^{-8}$ tesla

31. Suppose the *yz*-plane forms a chargeless boundary between two media of permittivities $\varepsilon_{\text{left}}$ and $\varepsilon_{\text{right}}$ where $\varepsilon_{\text{left}}:\varepsilon_{\text{right}}=1:2$. If the uniform electric field on the left is $\vec{E}_{left}=c(\hat{i}+\hat{j}+\hat{k})$ (where *c* is a constannt ), then the electric field on the right $\vec{E}_{right}$ is

(a) $c(2\hat{i}+\hat{j}+\hat{k})$ (b) $c(\hat{i}+2\hat{j}+2\hat{k})$ (c) $c\left(\frac{1}{2}\hat{i}+\hat{j}+\hat{k}\right)$ (d) $c\left(\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}\right)$

32. A porton moves with a speed of 300 m/s in a circular orbit in the *xy*-plane in a magnetic field 1 tesla along the positive *z*-direction. When an electric field of 1 V/m is applied along the positive y-direction, the center of the circular orbit

(a) remains stationary

(b) moves at 1 m/s along the negative *x*-direction

(c) moves at 1 m/s along the positive *z*-direction

(d) moves at 1 m/s along the positive $x$-direction

33. Which of the following transformations $(V, \vec{A}) \to (V', \vec{A}')$ of the electrostatic potential $V$ and the vector potential $\vec{A}$ is a gauge transformation?

(a) $\left(V' = V + ax, \vec{A}' = \vec{A} + at\,\hat{k}\right)$ (b) $\left(V' = V + ax, \vec{A}' = \vec{A} - at\,\hat{k}\right)$

(c) $\left(V' = V + ax, \vec{A}' = \vec{A} + at\,\hat{i}\right)$ (d) $\left(V' = V + ax, \vec{A}' = \vec{A} - at\,\hat{i}\right)$

34. The ratio of the energy of the first excited state $E_1$, to that of the ground state $E_0$ of a particle in a three-dimensional rectangular box of sides $L$, $L$ and $L/2$ is

(a) 3:2 (b) 2:1 (c) 4:1 (d) 4:3

35. The wavefunction of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p) = \int \psi(x) e^{-ipx/h} dx$ in the momentum representation. If the action of an operator $\hat{T}$ on $\psi(x)$ is given by $\hat{T}\psi(x) = \psi(x+a)$, where $a$ is a constant, then $\hat{T}\phi(p)$ is given by

(a) $-\frac{i}{h} ap\phi(p)$ (b) $e^{-iap/h}\phi(p)$ (c) $e^{+iap/h}\phi(p)$ (d) $\left(1 + \frac{i}{h} ap\right)\phi(p)$

36. If $L_i$ are the components of the angular momentum operator $\vec{L}$, then the operator $\sum_{i=1,2,3}\left[\left[\vec{L}, L_i\right], L_i\right]$ equals

(a) $\vec{L}$ (b) $2\vec{L}$ (c) $3\vec{L}$ (d) $-\vec{L}$

37. A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where $k(t)$ is a time dependent parameter. Then $\frac{d}{dt}\langle V \rangle$, the rate of change of the expectation value $\langle V \rangle$ of the potential energy, is

(a) $\frac{1}{2}\frac{dk}{dt}\langle x^2 \rangle + \frac{k}{2m}\langle xp + px \rangle$ (b) $\frac{1}{2}\frac{dk}{dt}\langle x^2 \rangle + \frac{k}{2m}\langle p^2 \rangle$

(c) $\frac{k}{2m}\langle xp + px \rangle$ (d) $\frac{1}{2}\frac{dk}{dt}\langle x^2 \rangle$

38. A system of $N$ distinguishable particles, each of which can be in one of the two energy levels 0 and $\varepsilon$, has a total energy $n\varepsilon$, where $n$ is an interger. The entropy of the system is proportional to

(a) $N \ell n\, n$ (b) $n \ell n\, N$ (c) $\ell n\left(\frac{N!}{n!}\right)$ (d) $\ell n\left(\frac{N!}{n!(N-n)!}\right)$

39. A system of $N$ non-intacting classical particles, each of mass $m$ is in a two-dimensional harmonic potential of the form $V(r) = \alpha\left(x^2 + y^2\right)$ where $\alpha$ is a positive constant. The canonical partition

function of the system at temperature $T$ is $\left(\beta = \dfrac{1}{k_B T}\right)$.

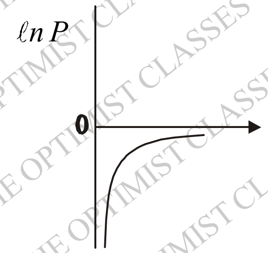
(a) $\left[\left(\dfrac{\alpha}{2m}\right)^2 \dfrac{\pi}{\beta}\right]^N$ (b) $\left(\dfrac{2m\pi}{\alpha\beta}\right)^{2N}$ (c) $\left(\dfrac{\alpha\pi}{2m\beta}\right)^N$ (d) $\left(\dfrac{2m\pi^2}{\alpha\beta^2}\right)^N$

40. In a two-state system, the transition rate of a particle from state 1 to state 2 is $t_{12}$, and the transition rate from state 2 to state 1 is $t_{21}$. In the steady state, the probability of finding the particle in state 1 is

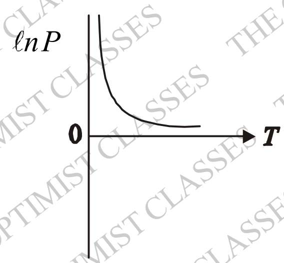
(a) $\dfrac{t_{21}}{t_{12}+t_{21}}$ (b) $\dfrac{t_{12}}{t_{12}+t_{21}}$ (c) $\dfrac{t_{12}t_{21}}{t_{12}+t_{21}}$ (d) $\dfrac{t_{12}-t_{21}}{t_{12}+t_{21}}$

41. The condition for the liquid and vapour phases of a fluid to be in equilibrium is given by the approximate equation $\dfrac{dP}{dT} \approx \dfrac{Q_\ell}{T v_{\text{vap}}}$ (Clausius-Clayperon equation), where $v_{\text{vap}}$ is the volume per particle in the vapour phase, and $Q_l$ is the latent heat, which may be taken to be a constant. If the vapour obeys ideal gas law, which of the following plots is correct?

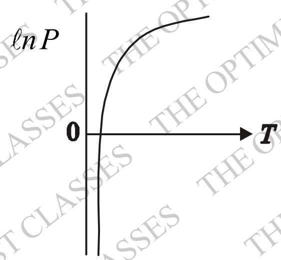
(a)



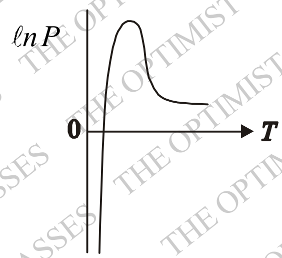

(b)



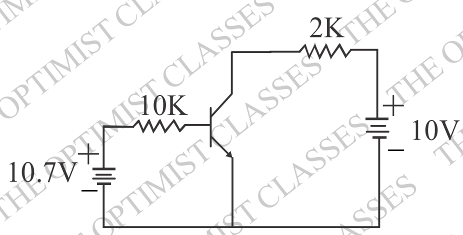

(c)



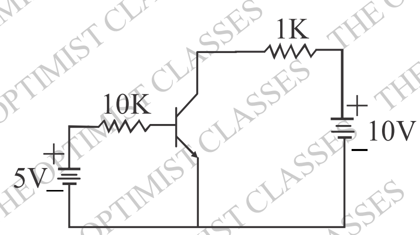

(d)




42. Consider the circuits shown in Figures (a) and (b) below.




(a)




(b)

If the transistors in Figures (a) and (b) have current gain $(\beta_{dc})$ of 100 and 10 respectively, then they operate in the

(a) active region and saturation region respectively

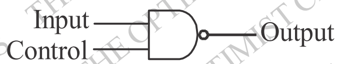
(b) saturation region and active region respectively

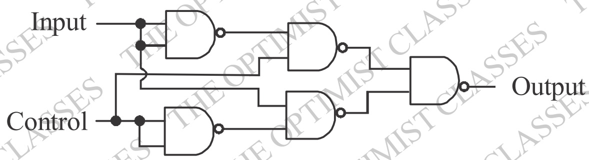
(c) saturation region in both cases
(d) active region in both cases

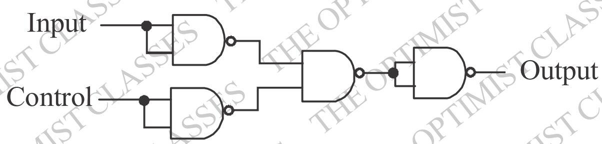
43. The viscosity $\eta$ of a liquid is given by Poiseuille's formula $\eta = \frac{\pi P a^4}{8lV}$. Assume that $l$ and $V$ can be measured very accurately, but the pressure $P$ has an rms errors of 1% and the radius $a$ has independent rms error of 3%. The rms errors of the viscosity is closest to

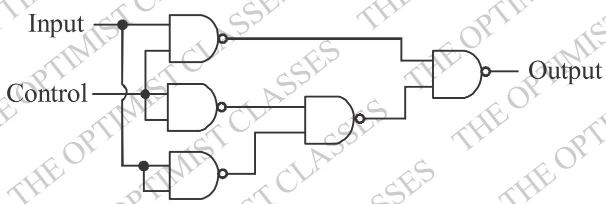
(a) 2% (b) 4% (c) 12% (d) 13%

44. Which of the following circuits behaves as a controlled inverter ?

(a)



(b)



(c)



(d)



45. The concentration of electrons, $n$ and holes, $p$ for an intrinsic semiconductor at a temperature $T$ can be expressed as $n = p = AT^{3/2}\exp\left(-\frac{E_g}{2k_BT}\right)$, where $E_g$ is the band gap and $A$ is a constant. If the mobility of both types of carries is proportional to $T^{-3/2}$, then the log of conductivity is a linear function of $T^{-1}$ with slope

(a) $\frac{E_g}{(2k_B)}$ (b) $\frac{E_g}{k_B}$ (c) $\frac{-E_g}{(2k_B)}$ (d) $\frac{-E_g}{k_B}$

**Part - C**

46. Three real variables $a$, $b$ and $c$ are each randomly chosen from a uniform probability distribution in the interval $[0,1]$. The probability that $a + b > 2c$ is

(a) 3/4 (b) 2/3 (c) 1/2 (d) 1/4

47. The rank-2 tensor $x_i x_j$, where $x_i$ are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements of each set transform only into linear combinations of elements in that set) containing

(a) 4 and 2 elements (b) 5 and 1 elements
(c) 3, 2 and 1 elements (d) 4, 1 and 1 elements

48. Consider the differential equation $\frac{dy}{dx} = x^2 - y$ with the initial condition $y = 2$ at $x = 0$. Let $y_{(1)}$ and $y_{(1/2)}$ be the solutions at $x = 1$ obtained using Euler's forward algorithm with step size 1 and 1/2 respectively.

The value of $\left(y_{(1)} - y_{(1/2)}\right) / y_{(1/2)}$ is

(a) $-\frac{1}{2}$ (b) $-1$ (c) $\frac{1}{2}$ (d) 1

49. Let $f(x,t)$ be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1-dimension. If at $t = 0$, $f(x,0) = e^{-x^2}$ and $\frac{\partial f}{\partial t}(x,0) = 0$ for all $x$, then $f(x,t)$ for all future times $t > 0$ is described by

(a) $e^{-\left(x^2 - v^2 t^2\right)}$ (b) $e^{-(x-vt)^2}$

(c) $\frac{1}{4} e^{-(x-vt)^2} + \frac{3}{4} e^{-(x+vt)^2}$ (d) $\frac{1}{2}\left[e^{-(x-vt)^2} + e^{-(x+vt)^2}\right]$

50. Let $q$ and $p$ be the canonical coordinate and momentum of a dynamical system. Which of the following transformations is canonical?

$$A : Q_1 = \frac{1}{\sqrt{2}} q^2 \quad \text{and} \quad P_1 = \frac{1}{\sqrt{2}} p^2$$

$$B ; Q_2 = \frac{1}{\sqrt{2}}(p+q) \text{ and } p_2 = \frac{1}{\sqrt{2}}(p-q)$$

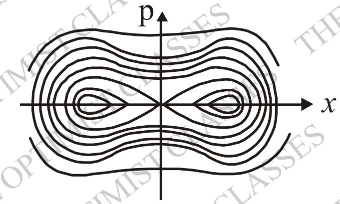
(a) neither A nor B (b) both A and B (c) only A (b) only B

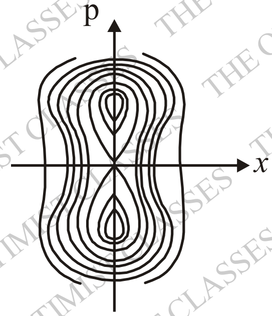
51. The differential cross-section for scattering by a target is given by $\frac{d\sigma(\theta,\phi)}{d\Omega} = a^2 + b^2 \cos^2\theta$. If $N$ is the flux of the incoming particles, the number of particles scattered per unit time is

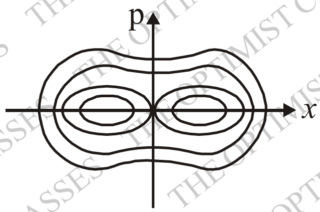
(a) $\frac{4\pi}{3} N\left(a^2 + b^2\right)$ (b) $4\pi N\left(a^2 + \frac{1}{6} b^2\right)$

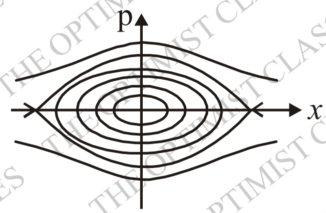
(c) $4\pi N\left(\frac{1}{2} a^2 + \frac{1}{3} b^2\right)$ (d) $4\pi N\left(a^2 + \frac{1}{3} b^2\right)$

52. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional potential $V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$ ?

(a) 


(b) 


(c) 


(d) 


53. Consider a rectangular wave guide with transverse dimensions $2\,\text{m} \times 1\,\text{m}$ driven with an angular frequency $\omega = 10^9$ rad/s. Which transverse electric (TE) modes will propagate in this wave guide?

(a) $TE_{10}, TE_{01}$ and $TE_{20}$
(b) $TE_{10}, TE_{11}$ and $TE_{20}$
(c) $TE_{01}, TE_{10}$ and $TE_{11}$
(d) $TE_{01}, TE_{10}$ and $TE_{22}$

54. A rod of length $L$ carries a total charge $Q$ distributed uniformly. If this is observed in a frame moving with a speed $v$ along the rod, the charge per unit length (as measured by the moving observer) is

(a) $\frac{Q}{L}\left(1-\frac{v^2}{c^2}\right)$
(b) $\frac{Q}{L}\sqrt{1-\frac{v^2}{c^2}}$
(c) $\frac{Q}{L\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$
(d) $\frac{Q}{L\left(1-\frac{v^2}{c^2}\right)}$

55. The electric and magnetic fields in the charge free region z > 0 are given by

$$\vec{E}(\vec{r},t) = E_0 e^{-k_1 z}\cos(k_2 x - \omega t)\,\hat{j}$$

$$\vec{B}(\vec{r},t) = \frac{E_0}{\omega} e^{-k_1 z}\left[k_1 \sin(k_2 x - \omega t)\,\hat{i} + k_2 \cos(k_2 x - \omega t)\,\hat{k}\right]$$

Where $\omega$, $k_1$ & $k_2$ are positive constants. The average energy flow in the $x$-direction is

(a) $\frac{E_0^2 k_2}{2\mu_0 \omega} e^{-2k_1 z}$
(b) $\frac{E_0^2 k_2}{\mu_0 \omega} e^{-2k_1 z}$
(c) $\frac{E_0^2 k_1}{2\mu_0 \omega} e^{-2k_1 z}$
(d) $\frac{1}{2} c\varepsilon_0 E_0^2 e^{-2k_1 z}$

56. A uniform magnetic field in the positive $z$-direction passes through a circular wire loop of radius 1 cm and resistenace $1\Omega$ lying in the $xy$-plane. The field strength is reduced from 10 tesla to 9 tesla in 1 s. The charge transferred across any point in the wire is approximately

(a) $3.1\times10^{-4}$ coulomb
(b) $3.4\times10^{-4}$ coulomb
(c) $4.2\times10^{-4}$ coulomb
(d) $5.2\times10^{-4}$ coulomb

57. The Dirac Hamiltonian $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$ for a free electron corresponds to the classical relation $E^2 = p^2c^2 + m^2c^4$. The classical energy-momentum relation of a particle of charge $q$ in a electromag

netic potential $(\phi, \vec{A})$ is $(E-q\phi)^2 = c^2\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2 + m^2c^4$. Therefore, the Dirac Hamiltonian for an electron in an electromagnetic field is

(a) $c\vec{\alpha}\cdot\vec{p}+\frac{e}{c}\vec{A}.\vec{A}+\beta mc^2 - e\phi$ (b) $c\vec{\alpha}\cdot\left(\vec{p}+\frac{e}{c}\vec{A}\right)+\beta mc^2 + e\phi$

(c) $c\left(\vec{\alpha}\cdot\vec{p}+e\phi+\frac{e}{c}\left|\vec{A}\right|\right)+\beta mc^2$ (d) $c\vec{\alpha}\cdot\left(\vec{p}+\frac{e}{c}\vec{A}\right)+\beta mc^2 - e\phi$

58. A particle mass $m$ is in a potential $V=\frac{1}{2}m\omega^2x^2$, where $\omega$ is a constant. Let $\hat{a}=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}+\frac{i\hat{p}}{m\omega}\right)$. In the Heisenberg picture $\frac{d\hat{a}}{dt}$ is given by

(a) $\omega\hat{a}$ (b) $-i\omega\hat{a}$ (c) $\omega\hat{a}^\dagger$ (d) $i\omega\hat{a}^\dagger$

59. A particle of energy $E$ scatters off a repulsive spherical potential

$$V(r)=\begin{cases} V_0 & \text{for } r<a \\ 0 & \text{for } r\geq a \end{cases}$$

where $V_0$ and $a$ are positive constants. In the low energy limit, the total scattering cross-section is $\sigma = 4\pi a^2\left(\frac{1}{ka}\tanh\, ka-1\right)^2$, where $k^2=\frac{2m}{\hbar^2}(V_0-E)>0$. In the limit $V_0\to\infty$ the ratio of to the classical scattering cross-section off a sphere of radius $a$ is

(a) 4 (b) 3 (c) 1 (d) 1/2

60. Two different sets of orthogonal basis vectros $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ and $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ are given for a two-dimensional real vector space. The matrix representation of a linear operator $\hat{A}$ in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

(a) $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ (b) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$

61. A large number $N$ of Brownian particles in one-dimension start their diffusive motion from the origin at time $t=0$. The diffusion coefficients is $D$. The number of particles crossing a point at a distance $L$ from the origin, per unit time, depends on $L$ and time $t$ as

(a) $\frac{N}{\sqrt{4\pi Dt}}e^{-L^2/(4Dt)}$ (b) $\frac{NL}{\sqrt{4\pi Dt}}e^{-4Dt/L^2}$ (c) $\frac{N}{\sqrt{16\pi Dt^3}}e^{-L^2/(4Dt)}$ (d) $Ne^{-4Dt/L^2}$

62. Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength $J$. The partition function of the system at temperature $T$ is given by

$\left(\beta = \dfrac{1}{k_B T}\right)$:

(a) $2e^{3\beta J} + 6e^{-\beta J}$ (b) $2e^{-3\beta J} + 6e^{\beta J}$

(c) $2e^{3\beta J} + 6e^{-3\beta J} + 3e^{\beta J} + 3e^{-\beta J}$ (d) $\left(2\cosh\beta J\right)^3$

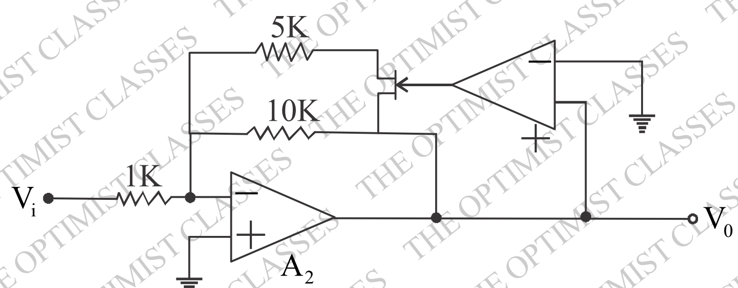
63. An ideal Bose gas in *d*-dimensions obeys the dispersion relation $\varepsilon\left(\vec{k}\right) = Ak^S$, where *A* and *s* are constants. For Bose-Einstein condensation to occurs, the occupancy of excited states

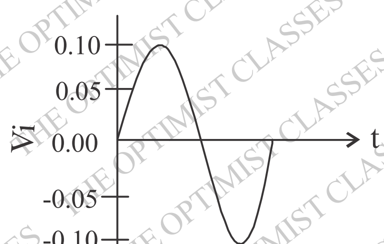
$$N_e = c\int_0^\infty \frac{\varepsilon^{(d-s)/s}}{e^{\beta(\varepsilon-\mu)} - 1}\, d\varepsilon$$

where *c* is a constant, should remain finite even for $\mu = 0$. This can happen if

(a) $\dfrac{d}{s} < \dfrac{1}{4}$ (b) $\dfrac{1}{4} < \dfrac{d}{s} < \dfrac{1}{2}$ (c) $\dfrac{d}{s} > 1$ (d) $\dfrac{1}{2} < \dfrac{d}{s} < 1$

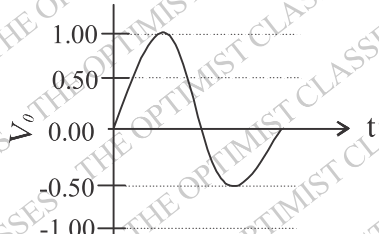
64. For the circuit and the input sinusodial waveform shown in the figures below, which is the correct waveform at the output?



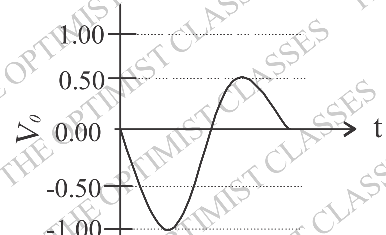



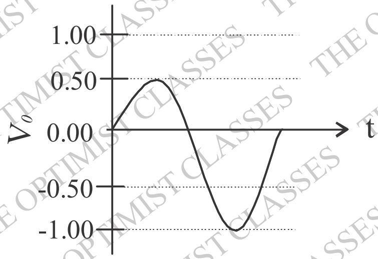

( The time scales in all the plots are the same).

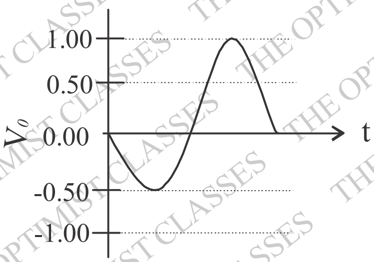
(a)



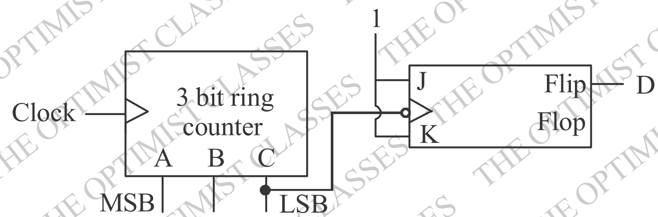

(b)



(c) 

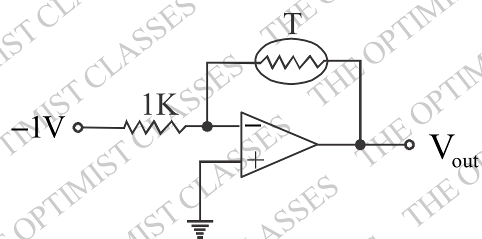
(d) 

65. For the logic circuit given below, the decimal count sequence and the modulus of the circuit corresponding to A B C D are




(a) $8 \to 4 \to 2 \to 1 \to 9 \to 5$ (mod 6)
(b) $8 \to 4 \to 2 \to 9 \to 5 \to 3$ (mod 6)
(c) $2 \to 5 \to 9 \to 1 \to 3$ (mod 5)
(d) $8 \to 5 \to 1 \to 3 \to 7$ (mod 5)

66. In the circuit given below, the thermistor has a resistance $3k\Omega$ at $25°C$. Its resistance decreases by $150\Omega$ per $°C$ upon heating. The output voltage of the circuit at $30°C$ is




(a) –3.75V (b) –2.25V (c) 2.25V (d) 3.75V

67. The low-energy electronic excitations in a two-dimensional sheet of graphene is given by $E(\vec{k}) = \hbar v k$, where $v$ is the velocity of the excitations. The density of states is proportional to

(a) $E$ (b) $E^{3/2}$ (c) $E^{1/2}$ (d) $E^2$

68. X-ray of wavelength $\lambda = a$ is reflected from the (1 1 1) plane of a simple cubic lattice. If the lattice constant is $a$, the corresponding Bragg angle (in radian) is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{8}$

69. The critical magnetic fields of a super-conductor at temperature 4*K* and 8 *K* are 11*mA/m* and 5.5*mA/m* respectively. The transition temperature is approximately

(a) 8.4*K* (b) 10.6*K* (c) 12.9*K* (d) 15.0*K*

70. A diatomic molecule has vibrational states with energies $E_v = \hbar\omega\left(v + \frac{1}{2}\right)$ and rotational states with energies $E_j = Bj(j+1)$, where $v$ and $j$ are non-negative integers. Consider the transitions in which

both the initial and final states are restricted to $v \le 1$ and $j \le 2$ and subject to the selection rules $\Delta v = \pm 1$ and $\Delta j = \pm 1$. Then the largest allowed energy of transition is

(a) $\hbar\omega - 3B$ (b) $\hbar\omega - B$ (c) $\hbar\omega + 4B$ (d) $2\hbar\omega + B$

71. Of the following term symbols of the $np^2$ atomic configurations, ${}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2$ and ${}^1D_2$, which is the grounded state?

(a) ${}^3P_0$ (b) ${}^1S_0$ (c) ${}^3P_2$ (d) ${}^3P_1$

72. A He-Ne laser operates by using two energy levels of Ne separated by $2.26eV$. Under steady state conditions of optical pumping, the equivalent temperature of the system at which the ratio of the number of atoms in the upper state to that in the lower state will be 1/20, is approximately (the Boltzmann constant $k_B = 8.6 \times 10^{-5}$ eV/ K ).

(a) $10^{10}$ K (b) $10^8$ K (c) $10^6$ K (d) $10^4$ K

73. Let us approximate the nuclear potential in the shell model by a 3-dimensional isotropic harmonic oscillator. Since the lowest two energy levels have angular momenta $l = 0$ and $l = 1$ respectively, which of the following two nuclei have magic numbers of protons and neutrons ?

(a) ${}^4_2$He and ${}^{16}_8$O (b) ${}^2_1$D and ${}^8_4$Be (c) ${}^4_2$He and ${}^8_4$Be (d) ${}^4_2$He and ${}^{12}_6$C

74. The Charm quark is assigned a charm quantum number $C = 1$. How should the Gellmann-Nishijima formula for electric charge be modified for four flavours of quarks?

(a) $I_3 + \frac{1}{2}(B - S - C)$ (b) $I_3 + \frac{1}{2}(B - S + C)$

(c) $I_3 + \frac{1}{2}(B + S - C)$ (d) $I_3 + \frac{1}{2}(B + S + C)$

75. The reaction ${}^2_1D + {}^2_1D \rightarrow {}^4_2He + \pi^0$ cannot proceed via strong interactions because it violates the conservation of

(a) angular momentum (b) electric charge (c) baryon number (d) isospin

## ANSWER KEY

| | | | | | | |
|---|---|---|---|---|---|---|
| 21. (c) | 22. (a) | 23. (b) | 24. (a) | 25. (a) | 26. (c) | 27. (b) |
| 28. (a) | 29. (b) | 30. (d) | 31. (c) | 32. (d) | 33. (d) | 34. (a) |
| 35. (c) | 36. (b) | 37. (a) | 38. (d) | 39. (d) | 40. (a) | 41. (a,c) |
| 42. (b) | 43. (c) | 44. (b,d) | 45. (c) | 46. (c) | 47. (b) | 48. (b) |
| 49. (d) | 50. (d) | 51. (d) | 52. (a) | 53. (a) | 54. (c) | 55. (a) |
| 56. (a) | 57. (d) | 58. (b) | 59. (a) | 60. (c) | 61. (a) | 62. (a) |
| 63. (c) | 64. (b) | 65. (b) | 66. (c) | 67. (a) | 68. (c) | 69. (b) |
| 70. (c) | 71. (a) | 72. (d) | 73. (a) | 74. (d) | 75. (d) | |