

# THE OPTIMIST CLASSES

## IIT-JAM TOPPERS



**8**  
AIR  
**MANOJ KUMAR SINGH**



**9**  
AIR  
**ABHAY**



**12**  
AIR  
**PAWAN**



**15**  
AIR  
**SATYAM**



**21**  
AIR  
**SOUMIL GIRISH SAHU**



**29**  
AIR  
**BHOOMIJA**



**31**  
AIR  
**AKSHIT AGGARWAL**



**32**  
AIR  
**SHIKHAR CHAMOLI**



**33**  
AIR  
**RAVI SINGH ADHIKARI**



**44**  
AIR  
**GAURAV JHA**



**56**  
AIR  
**SWAPNIL JOSHI**



**65**  
AIR  
**LOKESH BHATT**



**75**  
AIR  
**GOPESH VISHVAKARMA**



**77**  
AIR  
**VAIBHAV**



**80**  
AIR  
**SHASWAT CHAMOLI**

# CSIR-NET-JRF RESULTS 2022



**ANNU**  
DL01000308



**ALANKAR**  
UP15000162



**SAHIL RANA**  
HR09000108



**JAYESTHI**  
RJ11000161



**DASRATH**  
RJ06000682



**VIVEK**  
UK01000439



**UZAIR AHMED**  
UP02000246



**SURYA PRATAP SINGH**  
RJ06000232



**HIMANSHU**  
UP10000095



**CHANDAN**  
RJ09000159



**SAIKHOM JOHNSON**  
MN01000196



**AJAY SAINI**  
RJ06001744



**VIKAS YADAV**  
RJ06001102



**JYOTSNA KOHLI**  
UK02000262



**SHYAM SUNDAR**  
RJ06000615



# THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

CONTACT: 9871044043

CSIR-UGC-NET/JRF-June 2017

PREVIOUS YEAR QUESTION

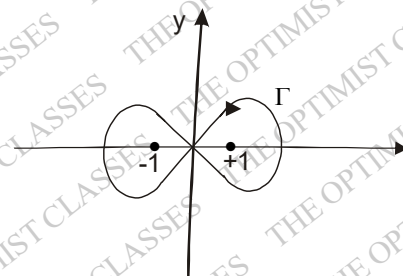
PHYSICAL SCIENCES

## Part - B

21. Which of the following cannot be eigenvalues of a real  $3 \times 3$  matrix  
 (a)  $2i, 0, -2i$  (b)  $1, 1, 1$  (c)  $e^{i\theta}, e^{-i\theta}, 1$  (d)  $i, 1, 0$
22. Let  $u(x, y) = e^{ax} \cos(by)$  be the real part of a function  $f(z) = u(x, y) + iv(x, y)$  of the complex variable  $z = x + iy$ , where  $a, b$  are real constants and  $a \neq 0$ . The function  $f(z)$  is complex analytic every where in the complex plane if and only if  
 (a)  $b = 0$  (b)  $b = \pm a$  (c)  $b = \pm 2\pi a$  (d)  $b = a \pm 2\pi$

23. The integral  $\oint_{\Gamma} \frac{ze^{iz/2}}{z-1} dz$  along the closed contour  $\Gamma$  shown in the figure is

- (a) 0  
 (b)  $2\pi$   
 (c)  $-2\pi$   
 (d)  $4\pi i$



24. The function  $y(x)$  satisfies the differential equation  $x \frac{dy}{dx} + 2y = \frac{\cos \pi x}{x}$ . If  $y(1) = 1$ , the value of  $y(2)$  is  
 (a)  $\pi$  (b) 1 (c)  $1/2$  (d)  $1/4$

25. The random variable  $x$  ( $-\infty < x < \infty$ ) is distributed according to the normal distribution

$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$ . The probability density of the random variable  $y = x^2$  is

- (a)  $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \leq y < \infty$  (b)  $\frac{1}{2\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \leq y < \infty$

$$(c) \frac{1}{\sqrt{2\sigma^2}} e^{-\frac{y}{2\sigma^2}}, 0 \leq y < \infty$$

$$(d) \frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{\sigma^2}}, 0 \leq y < \infty$$

26. The Hamiltonian for a system described by the generalized coordinate  $x$  and generalized momentum  $p$  is

$$H = \alpha x^2 p + \frac{p^2}{2(1+2\beta x)} + \frac{1}{2} \omega^2 x^2$$

where  $\alpha, \beta$  and  $\omega$  are constant. The corresponding Lagrangian is

$$(a) \frac{1}{2} (\dot{x} - \alpha x^2) (1 + 2\beta x) - \frac{1}{2} \omega^2 x^2$$

$$(b) \frac{1}{2(1+2\beta x)} \dot{x}^2 - \frac{1}{2} \omega^2 x^2 - \alpha x^2 \dot{x}$$

$$(c) \frac{1}{2} (\dot{x}^2 - \alpha^2 x^2) (1 + 2\beta x) - \frac{1}{2} \omega^2 x^2$$

$$(d) \frac{1}{2(1+2\beta x)} \dot{x}^2 - \frac{1}{2} \omega^2 x^2 + \alpha x^2 \dot{x}$$

27. An inertial observer sees two events  $E_1$  and  $E_2$  happening at the same location but  $6 \mu\text{s}$  apart in time. Another observer moving with a constant velocity  $v$  (with respect to the first one) sees the same events to be  $9 \mu\text{s}$  apart. The spatial distance between the events, as measured by the second observer, is approximately

$$(a) 300 \text{ m}$$

$$(b) 1000 \text{ m}$$

$$(c) 2000 \text{ m}$$

$$(d) 2700 \text{ m}$$

28. A ball weighing 100gm, released from a height of 5 m, bounces perfectly elastically off a plate. The collision time between the ball and the plate is 0.5 s. The average force on the plate is approximately

$$(a) 3 \text{ N}$$

$$(b) 2 \text{ N}$$

$$(c) 5 \text{ N}$$

$$(d) 4 \text{ N}$$

29. A solid vertical rod, of length  $L$ , and cross-sectional area  $A$ , is made of a material of Young's modulus  $Y$ . The rod is loaded with a mass  $M$ , and as a result, extends by a small amount  $\Delta L$  in the equilibrium condition. The mass is then suddenly reduced to  $M/2$ . As a result the rod will undergo longitudinal oscillation with an angular frequency

30. If the root-mean-squared momentum of a particle in the ground state of a one-dimensional simple harmonic potential is  $p_0$ , then its root-mean-squared momentum in the first excited state is

$$(a) p_0 \sqrt{2}$$

$$(b) p_0 \sqrt{3}$$

$$(c) p_0 \sqrt{2/3}$$

$$(d) p_0 \sqrt{3/2}$$

31. Consider a potential barrier  $A$  of height  $V_0$  and width  $b$ , and another potential barrier  $B$  of height  $2V_0$  and the same width  $b$ . The ratio  $T_A/T_B$  of tunnelling probabilities  $T_A$  and  $T_B$ , through barrier  $A$  and  $B$  respectively, for a particle of energy  $V_0/100$  is best approximated by

$$(a) \exp \left[ \sqrt{1.99} - \sqrt{0.99} \sqrt{\frac{8mV_0 b^2}{\hbar^2}} \right]$$

$$(b) \exp \left[ \sqrt{1.98} - \sqrt{0.98} \sqrt{\frac{8mV_0 b^2}{\hbar^2}} \right]$$

$$(c) \exp \left[ \sqrt{2.99} - \sqrt{0.99} \sqrt{\frac{8mV_0 b^2}{\hbar^2}} \right]$$

$$(d) \exp \left[ \sqrt{2.98} - \sqrt{0.98} \sqrt{\frac{8mV_0 b^2}{\hbar^2}} \right]$$

32. A constant perturbation  $H'$  is applied to a system for time  $\Delta t$  (where  $H' \Delta t \ll \hbar$ ) leading to a

transition from a state with energy  $E_i$  to another with energy  $E_f$ . If the time of application is doubled, the probability of transition will be

- (a) unchanged (b) doubled (c) quadrupled (d) halved

33. The two vectors  $\begin{pmatrix} a \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} b \\ c \end{pmatrix}$  are orthogonal if

- (a)  $a = \pm 1, b = \pm 1/\sqrt{2}, c = \pm 1/\sqrt{2}$  (b)  $a = \pm 1, b = \pm 1/2, c = 0$   
 (c)  $a = \pm 1, b = 0, c = \pm 1$  (d)  $a = \pm 1, b = \pm 1/2, c = 1/2$

34. Two long hollow co-axial conducting cylinders of radii  $R_1$  and  $R_2$  ( $R_1 < R_2$ ) are placed in vacuum as shown in the figure below.



The inner cylinder carries a charge  $+\lambda$  per unit length and the outer cylinder carries a charge  $-\lambda$  per unit length. The electrostatic energy per unit length of this system is

- (a)  $\frac{\lambda^2}{\pi \epsilon_0} \ln\left(\frac{R_2}{R_1}\right)$  (b)  $\frac{\lambda^2}{4\pi \epsilon_0} \left(\frac{R_2^2}{R_1^2}\right)$  (c)  $\frac{\lambda^2}{4\pi \epsilon_0} \ln\left(\frac{R_2}{R_1}\right)$  (d)  $\frac{\lambda^2}{2\pi \epsilon_0} \ln\left(\frac{R_2}{R_1}\right)$

35. A set  $N$  concentric circular loops of wire, each carrying a steady current  $I$  in the same direction, is arranged in a plane. The radius of the first loop is  $r_1 = a$  and the radius of the  $n^{\text{th}}$  loop is given by  $r_n = n r_{n-1}$ . The magnitude  $B$  of the magnetic field at the centre of the circles in the limit  $N \rightarrow \infty$ , is

- (a)  $\frac{\mu_0 I (e^2 - 1)}{4\pi a}$  (b)  $\frac{\mu_0 I (e - 1)}{\pi a}$  (c)  $\frac{\mu_0 I (e^2 - 1)}{8a}$  (d)  $\frac{\mu_0 I (e - 1)}{2a}$

36. An electromagnetic wave (of wavelength  $\lambda_0$  in free space) travels through an absorbing medium with dielectric permittivity given by  $\epsilon = \epsilon_R + i\epsilon_I$ , where  $\frac{\epsilon_I}{\epsilon_R} = \sqrt{3}$ . If the skin depth is  $\frac{\lambda_0}{4\pi}$ , the ratio of the amplitude of electric field  $E$  to that of the magnetic field  $B$ , in the medium (in ohms) is

- (a)  $120\pi$  (b)  $377$  (c)  $30\sqrt{2}\pi$  (d)  $30\pi$

37. The vector potential  $\vec{A} = k e^{-at} r \hat{r}$ , (where  $a$  and  $k$  are constants) corresponding to an electromagnetic field is changed to  $\vec{A}' = -k e^{-at} r \hat{r}$ . This will be a gauge transformation if the corresponding change  $\phi' - \phi$  in the scalar potential is

- (a)  $akr^2 e^{-at}$  (b)  $2akr^2 e^{-at}$  (c)  $-akr^2 e^{-at}$  (d)  $-2akr^2 e^{-at}$

38. A thermodynamic function  $G(T, P, N) = U - TS + PV$  is given in terms of the internal energy  $U$ , temperature  $T$ , entropy  $S$ , pressure  $P$ , volume  $V$  and the number of particles  $N$ . Which of the following relations is



true? (In the following  $\mu$  is the chemical potential).

- (a)  $S = -\left.\frac{\partial G}{\partial T}\right|_{N,P}$       (b)  $S = \left.\frac{\partial G}{\partial T}\right|_{N,P}$       (c)  $V = -\left.\frac{\partial G}{\partial P}\right|_{N,T}$       (d)  $\mu = -\left.\frac{\partial G}{\partial N}\right|_{P,T}$

39. A box, separated by a movable wall, has two compartments filled by a monoatomic gas of  $\frac{C_p}{C_v} = \gamma$ .

Initially the volumes of the two compartments are equal, but the pressure are  $3p_0$  and  $p_0$  respectively. When the wall is allowed to move, the final pressure in the two compartments becomes equal. The final pressure is

- (a)  $\left(\frac{2}{3}\right)^\gamma p_0$       (b)  $3\left(\frac{2}{3}\right)^\gamma p_0$       (c)  $\frac{1}{2}\left(1+3^{1/\gamma}\right)^\gamma p_0$       (d)  $\left(\frac{3^{1/\gamma}}{1+3^{1/\gamma}}\right)^\gamma p_0$

40. A gas of photons inside a cavity of volume  $V$  is in equilibrium at temperature  $T$ . If the temperature of the cavity is changed to  $2T$ , the radiation pressure will change by a factor of

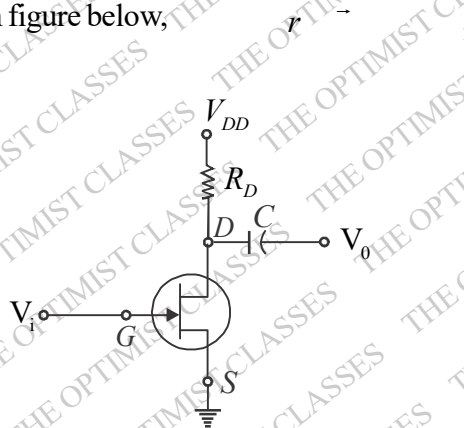
- (a) 2      (b) 16      (c) 8      (d) 4

41. In a thermodynamic system in equilibrium, each molecule can exist in three possible states with probabilities

$\frac{1}{2}, \frac{1}{3}$  and  $\frac{1}{6}$  respectively. The entropy per molecule is

- (a)  $k_B \ln 3$       (b)  $\frac{1}{2}k_B \ln 2 + \frac{2}{3}k_B \ln 3$   
 (c)  $\frac{2}{3}k_B \ln 2 + \frac{1}{2}k_B \ln 3$       (d)  $\frac{1}{2}k_B \ln 2 + \frac{1}{6}k_B \ln 3$

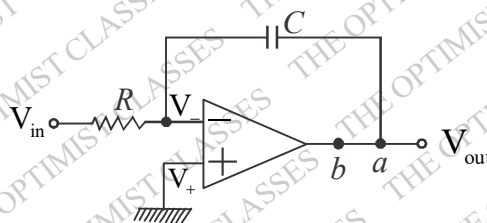
42. In the  $n$ -channel JFET shown in figure below,



If the drain  $D$ -source  $S$  saturation current  $I_{DSS}$  is 10 mA and the pinch-off voltage  $V_p$  is  $-8V$ , then the voltage across points  $D$  and  $S$  is

- (a)  $11.125V$       (b)  $10.375V$       (c)  $5.75V$       (d)  $4.75V$

43. The gain of the circuit given below is  $\frac{1}{\omega RC}$ .



The modification in the circuit required to introduce a dc feedback is to add a resistor

- (a) between  $a$  and  $b$  (b) between positive terminal of the op-amp and ground  
(c) in series with  $C$  (d) parallel to  $C$

44. A  $2 \times 4$  decoder with an enable input can function as a  
(a)  $4 \times 1$  multiplexer (b)  $1 \times 4$  demultiplexer  
(c)  $4 \times 2$  encoder (d)  $4 \times 2$  priority encoder
45. The experimentally measured values of the variables  $x$  and  $y$  are  $2.00 \pm 0.05$  and  $3.00 \pm 0.02$ , respectively. What is the error in the calculated value of  $z = 3y - 2x$  from the measurements?  
(a) 0.12 (b) 0.05 (c) 0.03 (d) 0.07

### Part - C

46. The Green's function satisfying  $\frac{d^2}{dx^2} g(x, x_0) = \delta(x - x_0)$ , with the boundary conditions is

(a)  $\begin{cases} \frac{1}{2L}(x_0 - L)(x + L), & -L \leq x < x_0 \\ \frac{1}{2L}(x_0 + L)(x - L), & x_0 \leq x \leq L \end{cases}$  (b)  $\begin{cases} \frac{1}{2L}(x_0 + L)(x + L), & -L \leq x < x_0 \\ \frac{1}{2L}(x_0 - L)(x - L), & x_0 \leq x \leq L \end{cases}$

(c)  $\begin{cases} \frac{1}{2L}(L - x_0)(x + L), & -L \leq x < x_0 \\ \frac{1}{2L}(x_0 + L)(L - x), & x_0 \leq x \leq L \end{cases}$  (d)  $\frac{1}{2L}(x - L)(x + L), -L \leq x < L$

47. Let  $\sigma_x, \sigma_y$  and  $\sigma_z$  be the pauli matrices and

$x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(\frac{i\theta\sigma_z}{2}\right) \times [x\sigma_x + y\sigma_y + z\sigma_z] \exp\left(\frac{-i\theta\sigma_z}{2}\right)$ . Then the coordinates are related as follows

(a)  $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  (b)  $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(c)  $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  (d)  $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

48. The interval  $[0, 1]$  is divided into  $2n$  parts of equal length to calculate the integral  $\int_0^1 e^{i2\pi x} dx$  using Simpson's

$\frac{1}{3}$ -rule. What is the minimum value of  $n$  for the result to be exact ?

- (a)  $\infty$  (b) 2 (c) 3 (d) 4

49. Which of the following sets of  $3 \times 3$  matrices ( in which  $a$  and  $b$  are real numbers) a group under matrix multiplication?

(a)  $\left\{ \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ b & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R} \right\}$

(b)  $\left\{ \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R} \right\}$

(c)  $\left\{ \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R} \right\}$

(d)  $\left\{ \begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R} \right\}$

50. The Lagrangian of a free relativistic particle (in one-dimension) of mass  $m$  is given by  $L = -m\sqrt{1 - \dot{x}^2}$ , where  $\dot{x} = dx/dt$ . If such a particle is acted upon by a constant force in the direction of its motion, the phase space trajectories obtained from the corresponding Hamiltonian are

- (a) ellipses (b) cycloids (c) hyperbolas (d) parabolas

51. A Hamiltonian system is described by the canonical coordinate  $q$  and canonical momentum  $p$ . A new coordinate  $Q$  is defined as  $Q(t) = q(t + \tau) + p(t + \tau)$ , where  $t$  is the time and  $\tau$  is a constant, that is, the new coordinate is a combination of the old coordinate and momentum at a shifted time. The new canonical momentum  $p(t)$  can be expressed as

(a)  $p(t + \tau) - q(t + \tau)$

(b)  $p(t + \tau) - q(t - \tau)$

(c)  $\frac{1}{2}[p(t - \tau) - q(t + \tau)]$

(d)  $\frac{1}{2}[p(t + \tau) - q(t + \tau)]$

52. The energy of a one-dimensional system, governed by the Lagrangian  $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^{2n}$ , where  $k$  and  $n$  are two positive constants, is  $E_0$ . The time period of oscillation  $\tau$  satisfies

(a)  $\tau \propto k^{-1/n}$

(b)  $\tau \propto k^{-1/2n} E_0^{1-n}$

(c)  $\tau \propto k^{-1/2n} E_0^{n-2}$

(d)  $\tau \propto k^{-1/n} E_0^{1+n}$

53. An electron is decelerated at a constant rate starting from an initial velocity  $u$  (where  $u \ll c$ ) to  $u/2$

during which it travels a distance  $s$ . The amount of energy lost to radiation is

(a)  $\frac{\mu_0 e^2 u^2}{3\pi m c^2 s}$

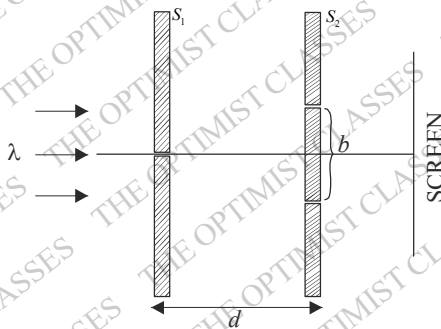
(b)  $\frac{\mu_0 e^2 u^2}{6\pi m c^2 s}$

(c)  $\frac{\mu_0 e^2 u}{8\pi m c s}$

(d)  $\frac{\mu_0 e^2 u}{16\pi m c s}$

54. The figure below describes the arrangement of slits and screens in a Young's double slit experiment. The width of the slit in  $S_1$  is  $a$  and the slits in  $S_2$  are of negligible width.

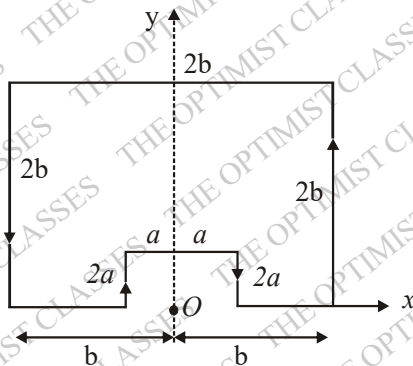




If the wavelength of the light is  $\lambda$ , the value of  $d$  for which the screen would be dark is

- (a)  $b\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$       (b)  $\frac{b}{2}\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$       (c)  $\frac{a}{2}\left(\frac{b}{\lambda}\right)^2$       (d)  $\frac{ab}{\lambda}$

55. A constant current  $I$  is flowing in a piece of wire that is bent into a loop as shown in the figure. The magnitude of the magnetic field at the point  $O$  is



- (a)  $\frac{\mu_0 I}{4\pi\sqrt{5}} \ln\left(\frac{a}{b}\right)$       (b)  $\frac{\mu_0 I}{4\pi\sqrt{5}} \left(\frac{1}{a} - \frac{1}{b}\right)$       (c)  $\frac{\mu_0 I}{4\pi\sqrt{5}} \left(\frac{1}{a}\right)$       (d)  $\frac{\mu_0 I}{4\pi\sqrt{5}} \left(\frac{1}{b}\right)$

56. Consider the potential  $V(\vec{r}) = \sum_i V_0 a^3 \delta^{(3)}(\vec{r} - \vec{r}_i)$ , where  $\vec{r}_i$  are the potential vectors of the vertices of a cube of length  $a$  centered at the origin and  $V_0$  is a constant. If  $V_0 a^2 \ll \frac{\hbar^2}{m}$ , the total scattering cross-section, in the low energy limit, is

- (a)  $16a^2 \left(\frac{mV_0 a^2}{\hbar^2}\right)$       (b)  $\frac{16a^2}{\pi^2} \left(\frac{mV_0 a^2}{\hbar^2}\right)^2$       (c)  $\frac{64a^2}{\pi} \left(\frac{mV_0 a^2}{\hbar^2}\right)^2$       (d)  $\frac{64a^2}{\pi^2} \left(\frac{mV_0 a^2}{\hbar^2}\right)^2$

57. The Coulomb potential  $V(r) = -e^2/r$  of a hydrogen atom is perturbed by adding  $H' = bx^2$  (where  $b$  is a constant) to the Hamiltonian. The first order correction to the ground state energy is (The ground state wavefunction is  $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ ).

- (a)  $2ba_0^2$       (b)  $2ba_0^2$       (c)  $ba_0^2/2$       (d)  $\sqrt{2}ba_0^2$

58. Using the trial function  $\psi(x) = \begin{cases} A(a^2 - x^2); & -a < x < a \\ 0 & ; \text{ otherwise} \end{cases}$  oscillator is

- (a)  $\hbar\omega$       (b)  $\sqrt{\frac{5}{14}}\hbar\omega$       (c)  $\frac{1}{2}\hbar\omega$       (d)  $\sqrt{\frac{5}{7}}\hbar\omega$

59. In the usual notation  $|nlm\rangle$  for the states of a hydrogen like atom, consider the spontaneous transitions  $|210\rangle \rightarrow |100\rangle$  and  $|310\rangle \rightarrow |100\rangle$ . If  $t_1$  and  $t_2$  are the lifetimes of the first and the second decaying states respectively, then the ratio  $t_1/t_2$  is proportional to

- (a)  $\left(\frac{32}{27}\right)^3$       (b)  $\left(\frac{27}{32}\right)^3$       (c)  $\left(\frac{2}{3}\right)^3$       (d)  $\left(\frac{3}{2}\right)^3$

60. A random variable  $n$  obeys poisson statistics. The probability of finding  $n = 0$  is  $10\%$ . The expectation value of  $n$  is nearest to

- (a) 14      (b)  $10^6$       (c)  $e$       (d)  $10^2$

61. The single particle energy levels of a non-interacting three-dimensional isotropic system, labelled by momentum  $k$ , are proportional to  $k^3$ . The ratio  $\bar{P}/\varepsilon$  of the average pressure  $\bar{P}$  to the energy density  $\varepsilon$  at a fixed temperature, is

- (a)  $1/3$       (b)  $2/3$       (c) 1      (d) 3

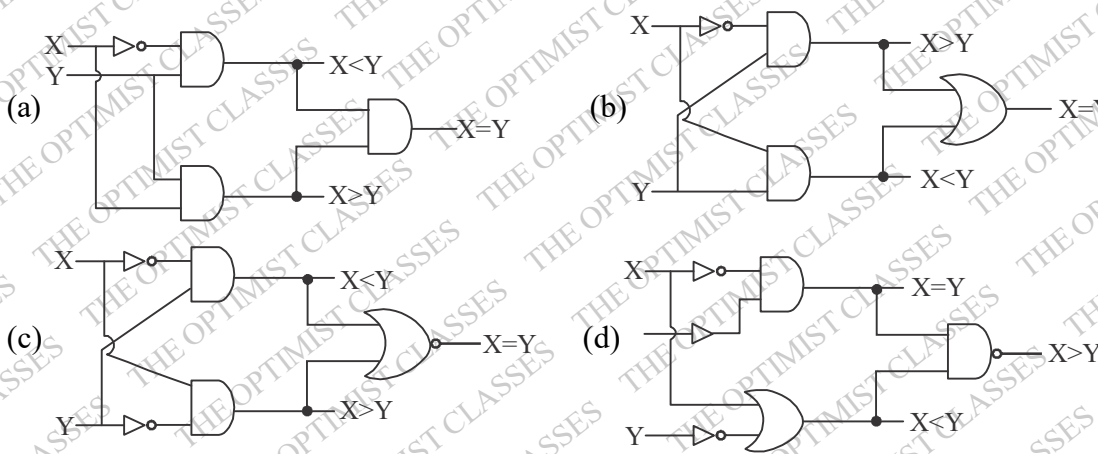
62. The Hamiltonian for three Ising spins  $S_0, S_1$  and  $S_2$  taking values  $\pm 1$  is  $H = -j S_0 (S_1 + S_2)$ . If the system is in equilibrium at temperature  $T$ , the average energy of the system, in terms of  $\beta = (k_B T)^{-1}$ , is

- (a)  $-\frac{1 + \cosh(2\beta j)}{2\beta \sinh(2\beta j)}$       (b)  $-2j [1 + \cosh(2\beta j)]$   
 (c)  $-2/\beta$       (d)  $-2j \frac{\sinh(2\beta j)}{1 + \cosh(2\beta j)}$

63. Let  $I_0$  be the saturation current,  $\eta$  the ideality factor and  $v_F$  and  $v_R$  the forward and reverse potentials, respectively, for a diode. The ratio  $R_R/R_F$  of its reverse and forward resistances  $R_R$  and  $R_F$  respectively, varies as (In the following  $k_B$  is the Boltzmann constant,  $T$  is the absolute temperature and  $q$  is the charge).

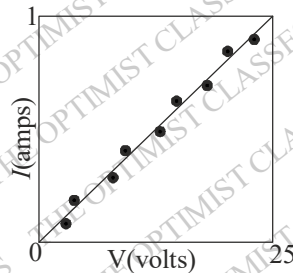
- (a)  $\frac{v_R}{v_F} \exp\left(\frac{qv_F}{\eta k_B T}\right)$       (b)  $\frac{v_F}{v_R} \exp\left(\frac{qv_F}{\eta k_B T}\right)$   
 (c)  $\frac{v_R}{v_F} \exp\left(-\frac{qv_F}{\eta k_B T}\right)$       (d)  $\frac{v_F}{v_R} \exp\left(-\frac{qv_F}{\eta k_B T}\right)$

64. In the figures below, X and Y are one bit inputs. The circuit which corresponds to a one bit comparator is



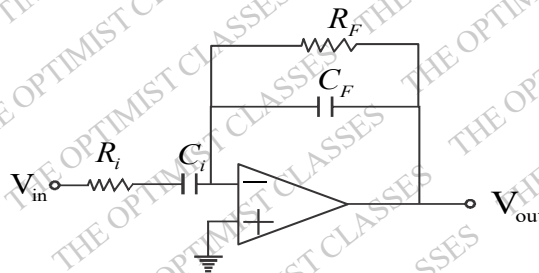
65. Both the data points and a linear fit to the current vs voltage of a resistor are shown in the graph below.

If the error in the slope is  $1.255 \times 10^{-3} \Omega^{-1}$ , then the value of resistance estimated from the graph is



- (a)  $(0.04 \pm 0.8) \Omega$     (b)  $(25.0 \pm 0.8) \Omega$     (c)  $(25 \pm 1.25) \Omega$     (d)  $(25 \pm 0.0125) \Omega$

66. In the following operational amplifier circuit  $C_{in} = 10 \text{ nF}$ ,  $R_{in} = 20 \text{ k}\Omega$ ,  $R_F = 200 \text{ k}\Omega$  and  $C_F = 100 \text{ pF}$ . The magnitude of the gain at a input signal frequency of 16 kHz is



- (a) 67    (b) 0.15    (c) 0.3    (d) 3.5

67. An atomic spectral line is observed to split into nine components due to Zeeman shift. If the upper state of the atom is  $^3D_2$  then the lower state will be

- (a)  $^3F_2$     (b)  $^3F_1$     (c)  $^3P_1$     (d)  $^3P_2$

68. If the coefficient of stimulated emission for a particular transition is  $2.1 \times 10^{19} \text{ m}^3 \text{ W}^{-1} \text{ s}^{-3}$  and the emitted photon is at wavelength  $3000 \text{ \AA}$ , then the lifetime of the excited state is approximately.



- (a) 20 ns                      (b) 40 ns                      (c) 80 ns                      (d) 100 ns

69. If the binding energies of the electron in the K and L shells of silver atom are 25.4 keV and 3.34 keV, respectively, then the kinetic energy of the Auger electron will be approximately
- (a) 22 keV                      (b) 9.3 keV                      (c) 10.5 keV                      (d) 18.7 keV
70. The energy gap and lattice constant of an indirect band gap semiconductor are 1.875 eV; 0.52 nm, respectively. For simplicity take the dielectric constant of the material to be unity. When it is excited by broadband radiation, an electron initially in the valence band at  $k = 0$  makes a transition to the conduction band. The wavevector of the electron in the conduction band, in terms of the wavevector  $k_{\max}$  at the edge of the Brillouin zone, after the transition is closest to
- (a)  $k_{\max} / 10$                       (b)  $k_{\max} / 100$                       (c)  $k_{\max} / 1000$                       (d) 0
71. The electrical conductivity of copper is approximately 95% of the electrical conductivity of silver, while the electron density in silver is approximately 70% of the electron density in copper. In Drude's model, the approximate ratio  $\tau_{Cu} / \tau_{Ag}$  of the mean collision time in copper ( $\tau_{Cu}$ ) to the mean collision time in silver ( $\tau_{Ag}$ ) is
- (a) 0.44                      (b) 1.50                      (c) 0.33                      (d) 0.66
72. The charge distribution inside a material of conductivity  $\sigma$  and permittivity  $\epsilon$  at initial time  $t = 0$  is  $\rho(r, 0) = \rho_0$ , constant. At subsequent times  $\rho(r, t)$  is given by
- (a)  $\rho_0 \exp\left(-\frac{\sigma t}{\epsilon}\right)$                       (b)  $\frac{1}{2} \rho_0 \left[1 + \exp\left(\frac{\sigma t}{\epsilon}\right)\right]$   
(c)  $\frac{\rho_0}{1 - \exp\left(\frac{\sigma t}{\epsilon}\right)}$                       (d)  $\rho_0 \cosh \frac{\sigma t}{\epsilon}$
73. If in a spontaneous  $\alpha$ -decay of  ${}^{232}_{92}\text{U}$  at rest, the total energy released in the reaction is  $Q$ , then the energy carried by the  $\alpha$ -particle is
- (a)  $\frac{57Q}{58}$                       (b)  $\frac{Q}{57}$                       (c)  $\frac{Q}{58}$                       (d)  $\frac{23Q}{58}$
74. The range of the nuclear force between two nucleons due to the exchange of pions is 1.40 fm. If the mass of the pion is  $140 \text{ MeV}/c^2$  and the mass of the rho-meson is  $770 \text{ MeV}/c^2$ , then the range of the force due to exchange of rho mesons is
- (a) 1.40 fm                      (b) 7.70 fm                      (c) 0.25 fm                      (d) 0.18 fm
75. A baryon  $X$  decays by strong interaction as  $X \rightarrow \Sigma^+ + \pi^- + \pi^0$ , where  $\Sigma^+$  is a member of the isotriplet  $(\Sigma^+, \Sigma^0, \Sigma^-)$ . The third component  $I_3$  of the isospin of  $X$  is
- (a) 0                      (b) 1/2                      (c) 1                      (d) 3/2

## ANSWER KEY

21. (d)	22. (b)	23. (c)	24. (d)	25. (a)	26. (a)	27. (c)
28. (c)	29. (a)	30. (b)	31. (a)	32. (c)	33. (c)	34. (c)
35. (d)	36. (d)	37. (c)	38. (a)	39. (c)	40. (b)	41. (c)
42. (d)	43. (d)	44. (b)	45. (a)	46. (a)	47. (b)	48. (b)
49. (c)	50. (c)	51. (d)	52. (b)	53. (d)	54. (b)	55. (b)
56. (c)	57. (b)	58. (b)	59. (a)	60. (a)	61. (c)	62. (d)
63. (b)	64. (c)	65. (b)	66. (d)	67. (c)	68. (c)	69. (d)
70. (c)	71. (d)	72. (a)	73. (a)	74. (c)	75. (a)	