

THE OPTIMIST CLASSES IIT-JAM TOPPERS

8
AIR



MANOJ KUMAR SINGH

9
AIR



ABHAY

12
AIR



PAWAN

15
AIR



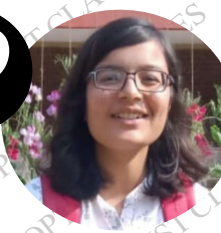
SATYAM

21
AIR



SOUMIL GIRISH SAHU

29
AIR



BHOOMIJA

31
AIR



AKSHIT AGGARWAL

32
AIR



SHIKHAR CHAMOLI

33
AIR



RAVI SINGH ADHIKARI

44
AIR



GAURAV JHA

56
AIR



SWAPNIL JOSHI

65
AIR



LOKESH BHATT

75
AIR



GOPESH VISHVAKARMA

77
AIR



VAIBHAV

80
AIR



SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022



ANNU
DL01000308



ALANKAR
UP15000162



SAHIL RANA
HR09000108



JAYESTHI
RJ11000161



DASRATH
RJ06000682



VIVEK
UK01000439



UZAIR AHMED
UP02000246



SURYA PRATAP SINGH
RJ06000232



HIMANSHU
UP10000095



CHANDAN
RJ09000159



SAIKHOM JOHNSON
MN01000196



AJAY SAINI
RJ06001744



VIKAS YADAV
RJ06001102



JYOTSNA KOHLI
UK02000262



SHYAM SUNDAR
RJ06000615

THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

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NET PAPER 2022

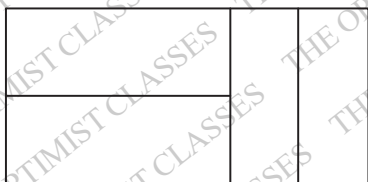
1. In a round-robin tournament, after each team has played exactly four matches, the number of wins/losses of 6 participating teams are as follows

Team	Win	Loss
A	4	0
B	0	4
C	3	1
D	2	2
E	0	4
F	3	1

Which of the two teams have certainly NOT played with each other?

- (a) A and B (b) C and F
(c) E and D (d) B and E

2. How many rectangles are there in the given figure?



- (a) 6 (b) 7
(c) 8 (d) 9

3. I have a brother who is 4 years elder to me, and a sister who was 5 years old when my brother was born. When my sister was born, my father was 24 years old. My mother was 27 years old when I was born. How old (in years) were my father and mother, respectively, when my brother was born?

- (a) 29 and 23 (b) 27 and 25
(c) 27 and 23 (d) 29 and 25

4. In a four-digit PIN, the third digit is the product of

the first two digits and the fourth digit is zero. The number of such PINs is

- (a) 42 (b) 41
(c) 40 (d) 39

5. A beam of square cross-section is to be cut of a wooden log. Assuming that the log is cylindrical, what approximately is the largest fraction of the wood by volume that can be fruitfully utilised as the beam?

- (a) 49% (b) 64%
(c) 71% (d) 81%

6. A vehicle has tyres of diameter $1m$ connected by a shaft directly to gearwheel A which meshes with gearwheel B as shown in the diagram. A has 12 teeth and B has 8. If points x on A and y on B are initially in contact, they will again be in contact after the vehicle has travelled a distance (in meters)



- (a) 2π (b) 3π
(c) 4π (d) 12π

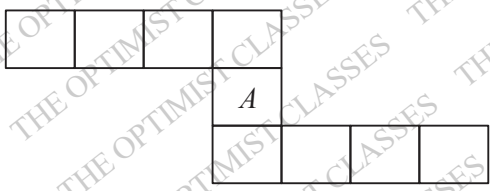
7. A plant grows by 10% of its height every three months. If the plant's height today is $1m$, its height after one year is the closest to

- (a) 1.10m (b) 1.21m
(c) 1.33m (d) 1.46m

8. If one letter each is drawn at random from the words CAUSE and EFFECT, the chance that they are the same is
- (a) $\frac{1}{30}$ (b) $\frac{1}{11}$
 (c) $\frac{1}{10}$ (d) $\frac{2}{11}$

9. After 12:00:00 the hour hand and minute hand of a clock will be perpendicular to each other for the first time at
- (a) 12:16:21 (b) 12:15:00
 (c) 13:22:21 (d) 12:48:08

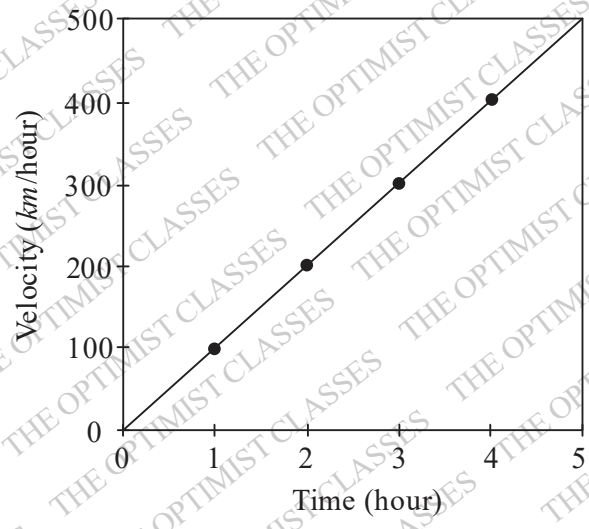
10. The squares in the following sketch are filled with digits 1 to 9, without any repetition, such that the numbers in the two horizontal rows add up to 20 each. What number appears in the square labelled A in the vertical column?



- (a) 3 (b) 5
 (c) 7
11. Tokens numbered from 1 to 25 are mixed and one token is drawn randomly. What is the probability that the number on the token drawn is divisible either by 4 or by 6?
- (a) $\frac{8}{25}$ (b) $\frac{10}{25}$
 (c) $\frac{9}{25}$ (d) $\frac{12}{25}$

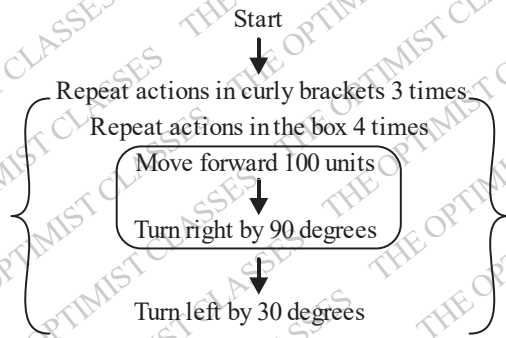
12. A boy has kites of which all but 9 are red, all but 9 are yellow, all but 9 are green, and all but 9 are blue. How many kites does he have?
- (a) 12 (b) 15
 (c) 9 (d) 18

13. Given plot describes the motion of an object with time.

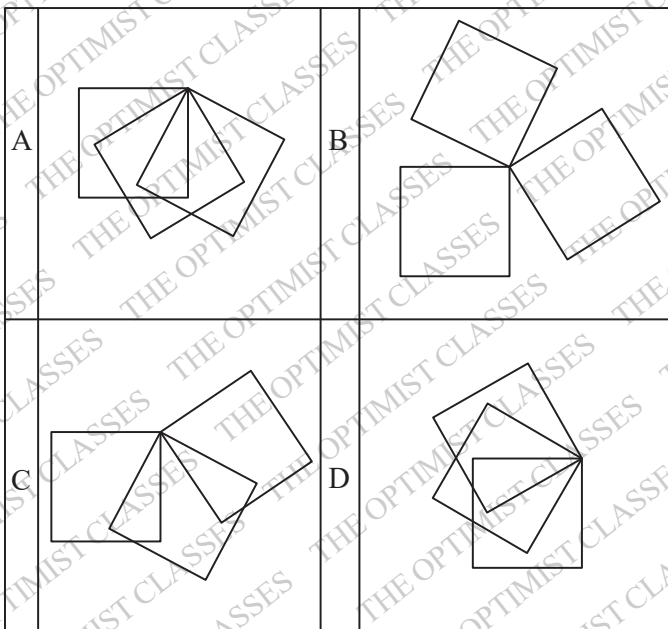


- Which one of the following statements is correct?
- (a) The object is moving with a constant velocity
 (b) The object covers equal distance every hour
 (c) The object is accelerating
 (d) Velocity of the object doubles every hour

14. A liar always lies and a non-liar, never. If in a group of n persons seated around a roundtable every-one calls his/her left neighbor a liar, then
- (a) all are liars
 (b) n must be even and every alternate person is a liar
 (c) n must be odd and every alternate person is a liar
 (d) n must be a prime
15. Starting from the top of a page and pointing downward, an ant moves according to the following commands



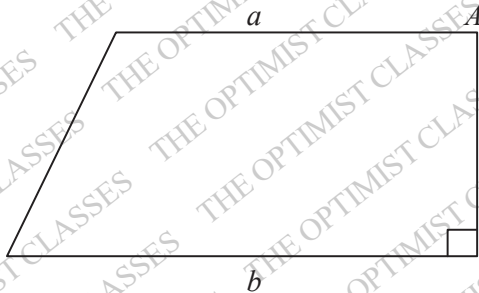
of the following paths



Which is the correct path of the ant?

- (a) A (b) B
(c) C (d) D

16. At what horizontal distance from A should a vertical line be drawn so as to divide the area of the trapezium shown in the figure into two equal parts? (a and b are lengths of the parallel sides)



- (a) $\frac{(a+b)}{4}$ (b) $\frac{(a+b)}{3}$
(c) $\frac{(a+b)}{2}$ (d) $\frac{(2a+b)}{2}$

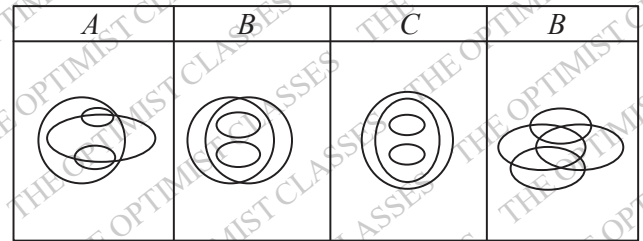
17. What is the product of the number of capital letters and the number of small letters of the English alphabet in the following text?

- (a) 17 (b) 37
(c) 53 (d) 63

18. On a track of $200m$ length, S runs from the starting point and R starts $20m$ ahead of S at the same time. Both reach the end of the track at the same time. S runs at a uniform speed of $10m/s$. If R also runs at a uniform speed, what is R 's speed (in m/s)?

- (a) 9 (b) 10
(c) 12 (d) 8

19. The correct pictorial representation of the relations among the categories PLAYERS, FEMALE CRICKETERS, MALE FOOTBALLERS and GRADUATES is



- (a) A (b) B
(c) C (d) D

20. Sections A , B , C and D of a class have 24, 27, 30 and 36 students, respectively. One section has boys and girls who are seated alternately in three rows, such that the first and the last positions in each row are occupied by boys. Which section could this be?

- (a) A (b) B
(c) C (d) D

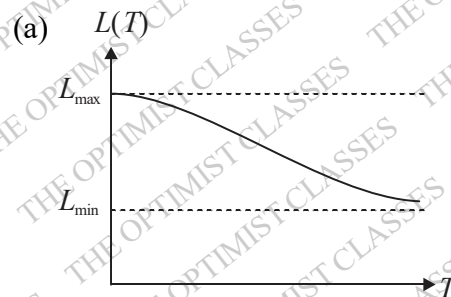
21. The periods of oscillation of a simple pendulum at the sea level and at the top of a mountain of height $6km$ are T_1 and T_2 , respectively. If the radius of

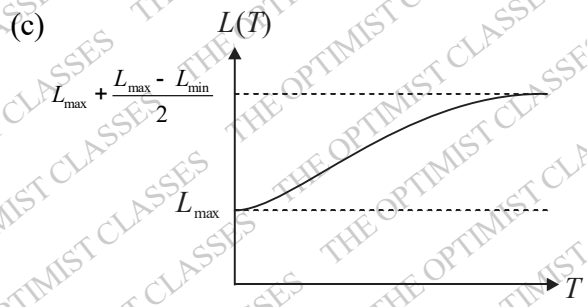
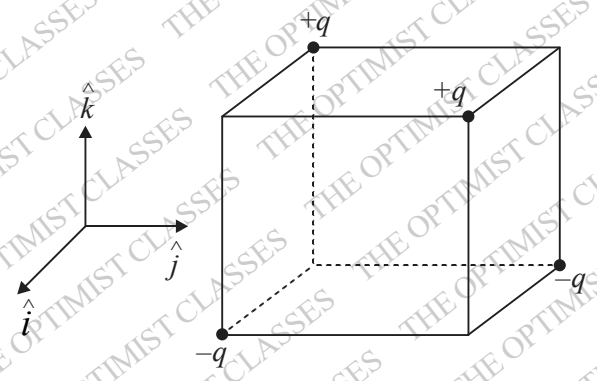
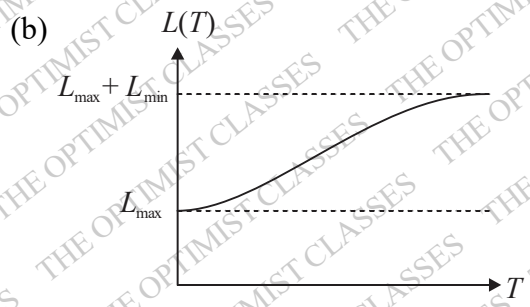
earth is approximately $6000km$, then $\frac{(T_2 - T_1)}{T_1}$ is

closest to

- (a) -10^{-4} (b) -10^{-3}
(c) 10^{-4} (d) 10^{-3}

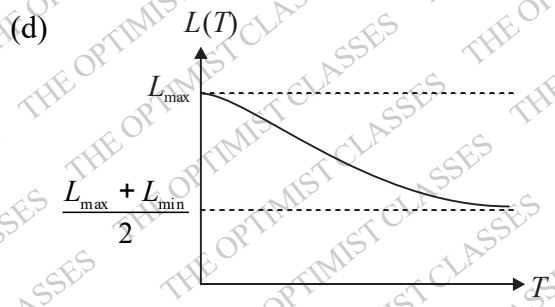
22. An elastic rod has a low energy state of length L_{\max} and high energy state of length L_{\min} . The best schematic representation of the temperature (T) dependence of the mean equilibrium length $L(T)$ of the rod, is





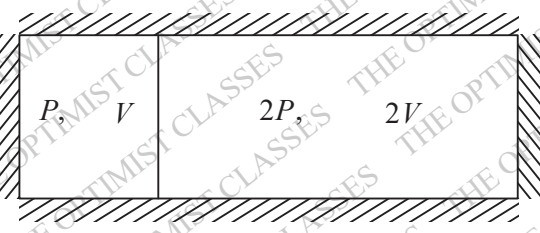
The electric dipole moment of this charge configuration is

- (a) $-2qa\hat{k}$ (b) $2qa\hat{k}$
 (c) $2qa(\hat{i} + \hat{j})$ (d) $2qa(\hat{i} - \hat{j})$



25.

A thermally isolated container, filled with an ideal gas at temperature T , is divided by a partition, which is clamped initially, as shown in the figure below



23. The electric and magnetic fields in an inertial frame

are $E = 3a\hat{i} - \hat{j}$ and $B = \frac{5a}{c}\hat{k}$, where a is a constant. A massive charged particle is released from rest. The necessary and sufficient condition that there is an inertial frame, where the trajectory of the particle is a uniform-pitched helix, is

- (a) $1 < a < \sqrt{2}$ (b) $-1 < a < 1$
 (c) $a^2 > 1$ (d) $a^2 > 2$

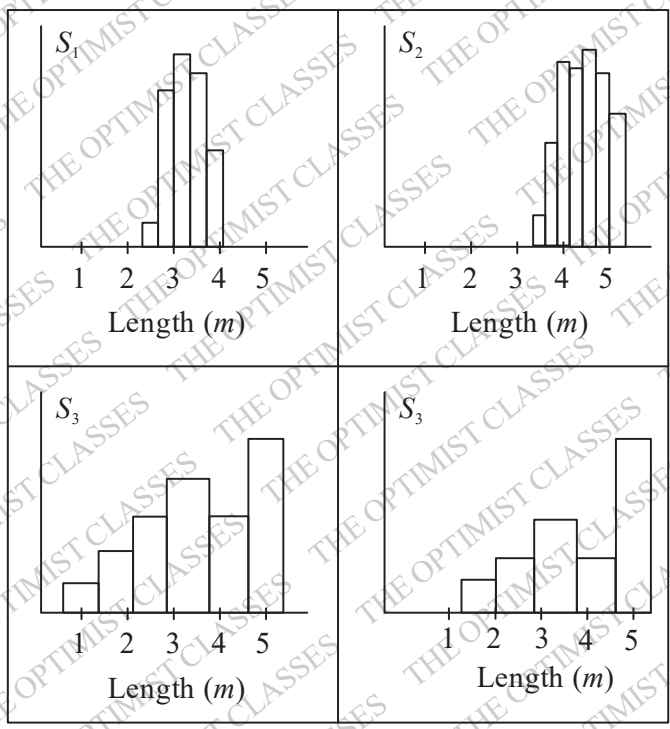
24. Two positive and two negative charges of magnitude q are placed on the alternate vertices of a cube of side a (as shown in the figure)

The partition does not allow the gas in the two parts to mix. It is subsequently released and allowed to move freely with negligible friction. The final pressure at equilibrium is

- (a) $\frac{5P}{3}$ (b) $\frac{5P}{4}$
 (c) $\frac{3P}{5}$ (d) $\frac{4P}{5}$

26.

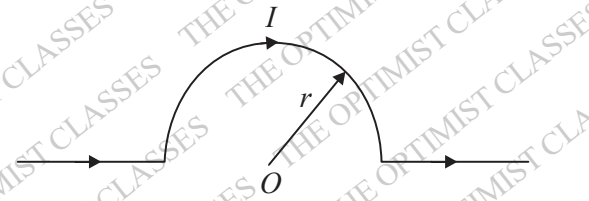
Four students (S_1, S_2, S_3 and S_4) make multiple measurements on the length of a table. The binned data are plotted as histograms in the following figure.



If the length of the table, specified by the manufacture, is $3m$, the student whose measurements have the minimum systematic error, is

- (a) S_2 (b) S_1
- (c) S_4 (d) S_3

27. A part of an infinitely long wire, carrying a current I , is bent in a semi circular arc of radius r (as shown in the figure)



The magnetic field at the centre O of the arc of

- (a) $\frac{\mu_0 I}{4r}$ (b) $\frac{\mu_0 I}{4\pi r}$
- (c) $\frac{\mu_0 I}{2r}$ (d) $\frac{\mu_0 I}{2\pi r}$

28. At $z=0$, the function $\frac{1}{z - \sin z}$ of a complex variable z has

- (a) no singularity (b) a simple pole
- (c) a pole of order 2 (d) a pole of order 3

29. The infinite series $\sum_{n=0}^{\infty} (n^2 + 3n + 2)x^n$ evaluated at $x = \frac{1}{2}$ is

(a) 16 (b) 32
 (c) 8 (d) 24

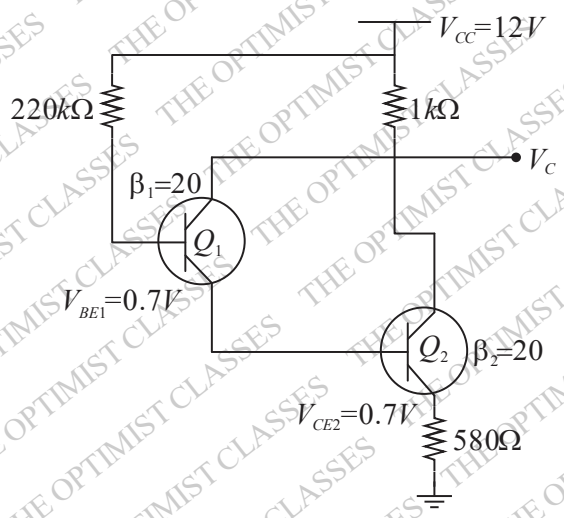
30. A particle of rest mass m is moving with a velocity $v\hat{k}$, with respect to an inertial frame S . The energy of the particle as measured by an observer S' , who is moving with a uniform velocity $u\hat{i}$ with respect to S (in terms of $\gamma_u = 1/\sqrt{1-u^2/c^2}$ and $\gamma_v = 1/\sqrt{1-v^2/c^2}$) is

- (a) $\gamma_u \gamma_v m (c^2 - uv)$
- (b) $\gamma_u \gamma_v m c^2$
- (c) $\frac{1}{2} (\gamma_u + \gamma_v) m c^2$
- (d) $\frac{1}{2} (\gamma_u + \gamma_v) m (c^2 - uv)$

31. If $Z = i^i$ (note that the exponent continues indefinitely), then a possible value of $\frac{1}{Z} \ln Z$ is

- (a) $2i \ln i$ (b) $\ln i$
- (c) $i \ln i$ (d) $2 \ln i$

32. The figure below shows a circuit with two transistors, Q_1 and Q_2 , having current gains β_1 and β_2 respectively

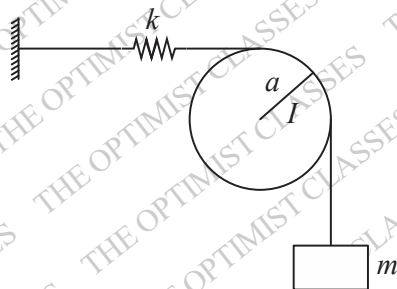


The collector voltage V_c will be closest to

- (a) $0.9V$ (b) $2.2V$
- (c) $2.9V$ (d) $4.2V$

33. A wire, connected to a massless spring of spring

constant k and a block of mass m , goes around a disc of radius a and moment of inertia I , as shown in the figure.



Assume that the spring remains horizontal, the pulley rotates freely and there is no slippage between the wire and the pulley. The angular frequency of small oscillations of the disc is

- (a) $\sqrt{\frac{2ka^2}{ma^2 + I}}$ (b) $\sqrt{\frac{ka^2}{ma^2 + I}}$
 (c) $\sqrt{\frac{ka^2}{ma^2 + 2I}}$ (d) $\sqrt{\frac{ka^2}{2ma^2 + I}}$

34. Consider the Hamiltonian $H = AI + B\sigma_x + C\sigma_y$, A, B and C are positive constants, I is the 2×2 identity matrix and σ_x, σ_y are Pauli matrices. If the normalized eigenvector corresponding to its

largest energy eigenvalue is $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ y \end{pmatrix}$, then y is

- (a) $\frac{B + iC}{\sqrt{B^2 + C^2}}$ (b) $\frac{A - iB}{\sqrt{A^2 + B^2}}$
 (c) $\frac{A - iC}{\sqrt{A^2 + C^2}}$ (d) $\frac{B - iC}{\sqrt{B^2 + C^2}}$

35. If the average energy $\langle E \rangle_T$ of a quantum harmonic oscillator at a temperature T is such that $\langle E \rangle_T = 2\langle E \rangle_{T \rightarrow 0}$, then T satisfies

- (a) $\coth\left(\frac{\hbar\omega}{k_B T}\right) = 2$ (b) $\coth\left(\frac{\hbar\omega}{2k_B T}\right) = 2$
 (c) $\coth\left(\frac{\hbar\omega}{k_B T}\right) = 4$ (d) $\coth\left(\frac{\hbar\omega}{2k_B T}\right) = 4$

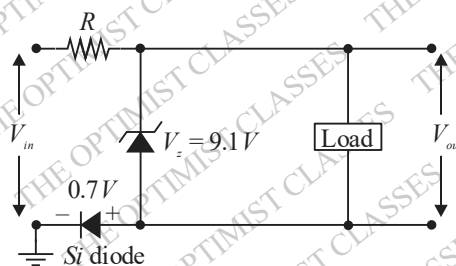
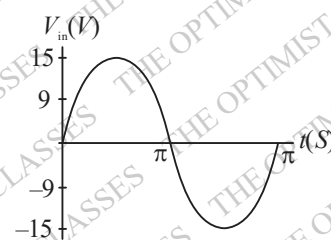
The Lagrangian of a system described by three generalized coordinates q_1, q_2 and q_3 is

$$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) + M\dot{q}_1\dot{q}_2 + kq_1q_3, \text{ where } m, M$$

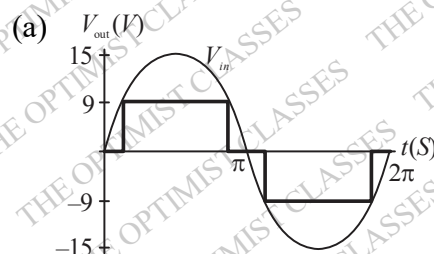
and k are positive constants. Then, as a function of time

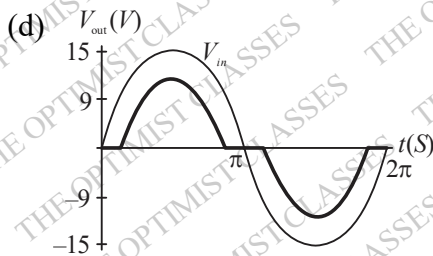
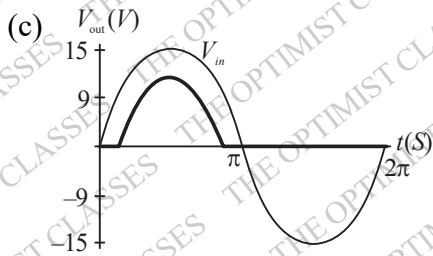
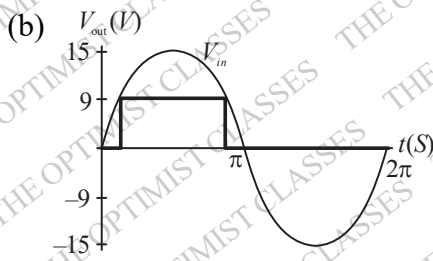
- (a) two coordinates remain constant and one evolves linearly
 (b) one coordinate remains constant, one evolves linearly and the third evolves as a quadratic function
 (c) one coordinate evolves linearly and two evolve quadratically
 (d) all three evolve linearly

A high impedance load (network) is connected in the circuit as shown below



The forward voltage drop for silicon diode is $0.7V$ and the Zener voltage is $9.10V$. If the input voltage (V_{in}) is sine wave with an amplitude of $15V$ (as shown in the figure above), which of the following waveform qualitatively describes the output voltage (V_{out}) across the load?





38. The value of the integral $\int_0^{\infty} dx e^{-x^{2m}}$, where m is a positive integer, is

(a) $\Gamma\left(\frac{m+1}{2m}\right)$ (b) $\Gamma\left(\frac{m-1}{2m}\right)$

(c) $\Gamma\left(\frac{2m+1}{2m}\right)$ (d) $\Gamma\left(\frac{2m-1}{2m}\right)$

39. If the expectation value of the momentum of a particle in one dimension is zero, then its (box-normalizable) wavefunction may be of the form

(a) $\sin kx$ (b) $e^{ikx} \sin kx$
 (c) $e^{ikx} \cos kx$ (d) $\sin kx + e^{ikx} \cos kx$

40. A walker takes steps, each of length L , randomly in the directions along east, west, north and south. After four steps its distance from the starting point is d . The probability that $d \leq 3L$ is

(a) $\frac{63}{64}$ (b) $\frac{59}{64}$

(c) $\frac{57}{64}$ (d) $\frac{55}{64}$

41. The momentum space representation of the Schrodinger equation of a particle in a potential $V(\vec{r})$ is

$$\left(|p|^2 + \beta(\nabla_p^2)^2\right)\psi(p,t) = i\hbar \frac{\partial}{\partial t}\psi(p,t), \text{ and } \beta$$

is a constant. The potential is (in the following V_0 and a are constants)

(a) $V_0 e^{-r^2/a^2}$ (b) $V_0 e^{-r^4/a^4}$

(c) $V_0 \left(\frac{r}{a}\right)^2$ (d) $V_0 \left(\frac{r}{a}\right)^4$

42. An electromagnetic wave is incident from vacuum normally on a planar surface of a non-magnetic medium. If the amplitude of the electric field of the incident wave is E_0 and that of the transmitted wave is $2E_0/3$, then neglecting any loss, the refractive index of the medium is

(a) 1.5 (b) 2.0
 (c) 2.4 (d) 2.7

43. Two $n \times n$ invertible real matrices A and B satisfy the relation $(AB)^T = -(A^{-1}B)^{-1}$

If B is orthogonal then A must be

(a) lower triangular (b) orthogonal
 (c) symmetric (d) antisymmetric

44. In terms of a complete set of orthonormal basis kets $|n\rangle$, $n = 0, \pm 1, \pm 2, \dots$, the Hamiltonian is

$$H = \sum_n (E|n\rangle\langle n| + \varepsilon|n+1\rangle\langle n+1| + \varepsilon\langle n+1|)$$

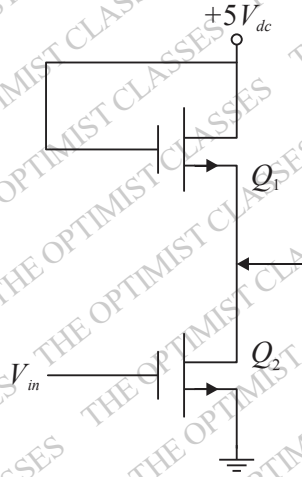
where E and ε are constants. The state

$$|\varphi\rangle = \sum_n e^{in\varphi}|n\rangle \text{ is an eigenstate with energy}$$

(a) $E + \varepsilon \cos \varphi$ (b) $E - \varepsilon \cos \varphi$

(c) $E + 2\varepsilon \cos \varphi$ (d) $E - 2\varepsilon \cos \varphi$

45. The circuit containing two n -channel MOSFETs shown below, works as



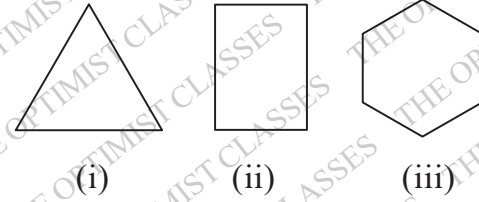
- (a) a buffer
 (b) a non-inverting amplifier
 (c) an inverter
 (d) a rectifier
46. Thermal neutrons may be detected most efficiently by a
 (a) ${}^6\text{Li}$ loaded plastic scintillator
 (b) Geiger-Muller counter
 (c) Inorganic scintillator CaF_2
 (d) Silicon detector
47. The energy levels of a system, which is in equilibrium at temperature $T = \frac{1}{(k_B\beta)}$, are $0, \varepsilon$ and 2ε . If two identical bosons occupy these energy levels, the probability of the total energy being 3ε , is

- (a) $\frac{e^{-\beta\varepsilon}}{1 + e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}}$
 (b) $\frac{e^{-3\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}}$
 (c) $\frac{e^{-3\beta\varepsilon}}{e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}}$
 (d) $\frac{e^{-3\beta\varepsilon}}{1 + e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}}$

48. The energies of a two-level system are $\pm E$. Consider an ensemble of such non-interacting systems at a temperature T . At low temperatures, the leading term in the specific heat depends on T as

- (a) $\frac{1}{T^2} e^{-E/k_B T}$ (b) $\frac{1}{T^2} e^{-2E/k_B T}$
 (c) $T^2 e^{-E/k_B T}$ (d) $T^2 e^{-2E/k_B T}$

49. The Figures (i), (ii) and (iii) below represent an equilateral triangle, a rectangle and a regular hexagon, respectively.



Which of these can be primitive unit cells of a Bravais lattice in two dimensions?

- (a) only (i) and (iii) but not (ii)
 (b) only (i) and (ii) but not (iii)
 (c) only (ii) and (iii) but not (i)
 (d) all of them

50. A receiver operating at 27°C has an input resistance of 100Ω . The input thermal noise voltage for this receiver with a bandwidth of 100kHz is closest to

- (a) 0.4nV (b) 0.6pV
 (c) 40mV (d) $0.4\mu\text{V}$

51. A stationary magnetic dipole $m = m\hat{k}$ is placed above an infinite surface ($z=0$) carrying a uniform surface current density $k = k\hat{i}$. The torque on the dipole is

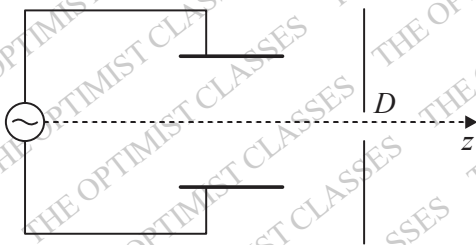
- (a) $\frac{\mu_0}{2} m k \hat{i}$ (b) $-\frac{\mu_0}{2} m k \hat{i}$
 (c) $\frac{\mu_0}{2} m k \hat{j}$ (d) $-\frac{\mu_0}{2} m k \hat{j}$

52. The tensor component of the nuclear force may be inferred from the fact that deuteron nucleus ${}^2_1\text{H}$

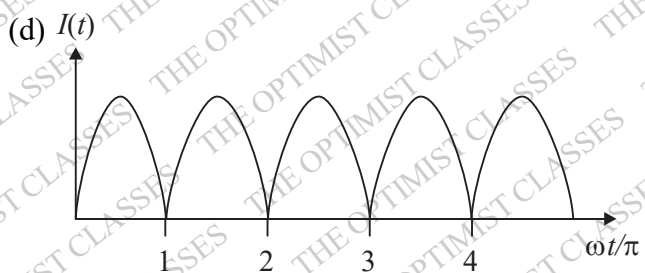
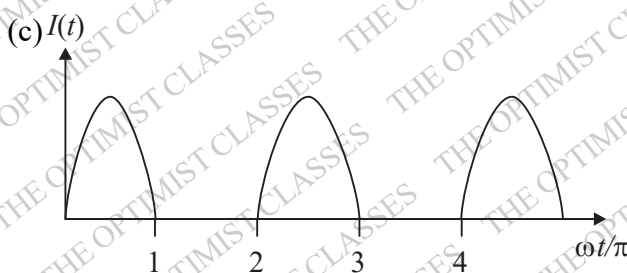
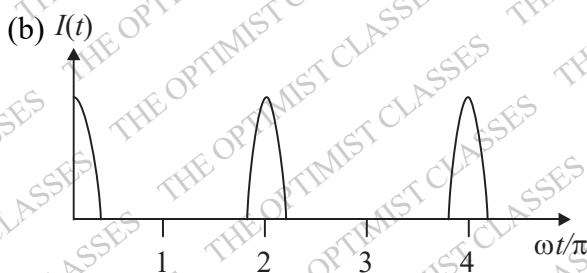
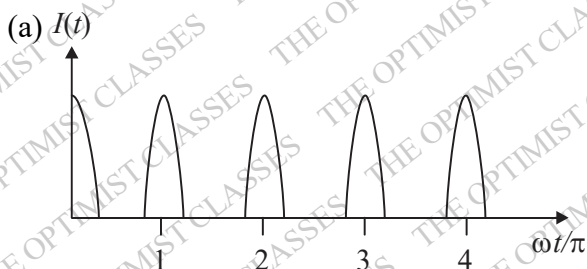
- (a) has only one bound state with total spin $s=1$
 (b) has a non-zero electric quadrupole moment in its ground state
 (c) is stable while triton ${}^3_1\text{H}$ is unstable
 (d) is the only two nucleon bound state

53. A high frequency voltage signal $V_i = V_m \sin \omega t$ is applied to a parallel plate deflector as shown in

the figure



An electron beam is passing through the deflector along the central line. The best qualitative representation of the intensity $I(t)$ of the beam after it goes through the narrow circular aperture D , is



54. At time $t = 0$, a particle is in the ground state of the Hamiltonian

$$H(t) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x \sin \frac{\omega t}{2} \quad \text{where } \lambda, \omega$$

and m are positive constants. To $O(\lambda^2)$, the probability that at $t = \frac{2\pi}{\omega}$, the particle would be in the

first excited state of $H(t=0)$ is

- (a) $\frac{9\lambda^2}{16m\hbar\omega^3}$ (b) $\frac{9\lambda^2}{8m\hbar\omega^3}$
 (c) $\frac{16\lambda^2}{9m\hbar\omega^3}$ (d) $\frac{8\lambda^2}{9m\hbar\omega^3}$

55. The electronic configuration of ^{12}C is $1s^2 2s^2 2p^2$.

Including LS coupling, the correct ordering of its energies is

- (a) $E(^3P_2) < E(^3P_1) < E(^3P_0) < E(^1D_2)$
 (b) $E(^3P_0) < E(^3P_1) < E(^3P_2) < E(^1D_2)$
 (c) $E(^1D_2) < E(^3P_2) < E(^3P_1) < E(^3P_0)$
 (d) $E(^3P_1) < E(^3P_0) < E(^3P_2) < E(^1D_2)$

56. The Laplace transform $L[f](y)$ of the function

$$f(x) = \begin{cases} 1 & \text{for } 2n \leq x \leq 2n+1 \\ 0 & \text{for } 2n+1 \leq x \leq 2n+2 \end{cases}, n=0,1,2,\dots$$

is

- (a) $\frac{e^{-y}(e^{-y}+1)}{y(e^{-2y}+1)}$ (b) $\frac{e^y - e^{-y}}{y}$
 (c) $\frac{e^y + e^{-y}}{y}$ (d) $\frac{e^y(e^y-1)}{y(e^{2y}-1)}$

57. The energy/ energies E of the bound state(s) of a particle of mass m in one dimension in the potential

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ -V_0, & 0 < x < a \quad (\text{where } V_0 > 0) \\ 0, & x \geq a \end{cases}$$

are determined by

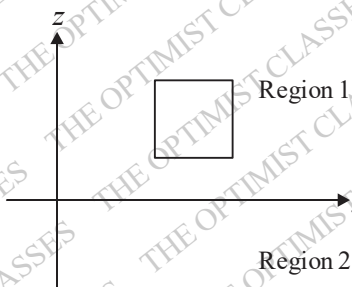
$$(a) \cot^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = \frac{E-V_0}{E}$$

$$(b) \tan^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = -\frac{E}{E+V_0}$$

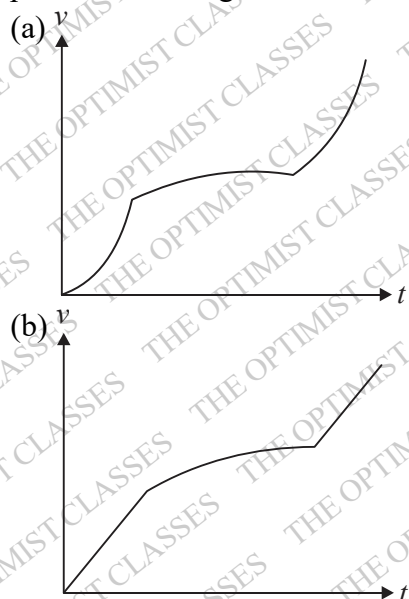
$$(c) \cot^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = -\frac{E}{E+V_0}$$

$$(d) \tan^2 \left(a \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \right) = \frac{E-V_0}{E}$$

58. A square conducting loop in the yz -plane, falls downward under gravity along the negative z -axis. Region 1, defined by $z > 0$ has a uniform magnetic field $B = B_0 \hat{i}$, while region 2 (defined by $z < 0$) has no magnetic field.



The time dependence of the speed $v(t)$ of the loop, as it starts to fall from well within the region 1 and passes into the region 2, is best represented by



59. The matrix corresponding to the differential operator $\left(1 + \frac{d}{dx}\right)$ in the space of polynomials of degree at most two, in the basis spanned by $f_1 = 1, f_2 = x$ and $f_3 = x^2$, is

$$(a) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(b) \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$(c) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

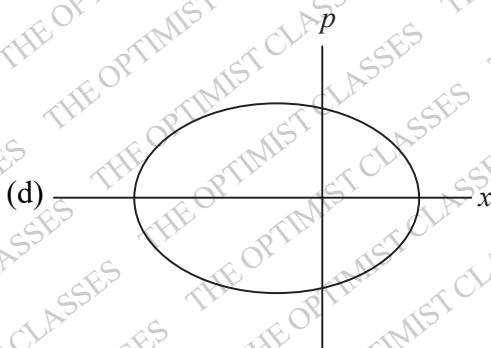
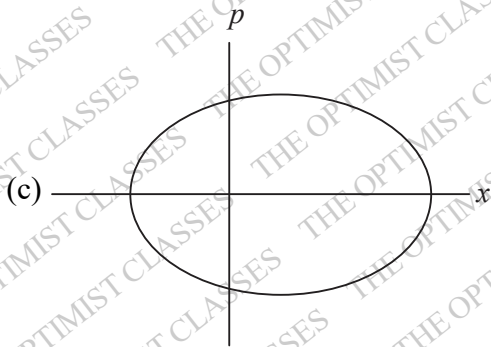
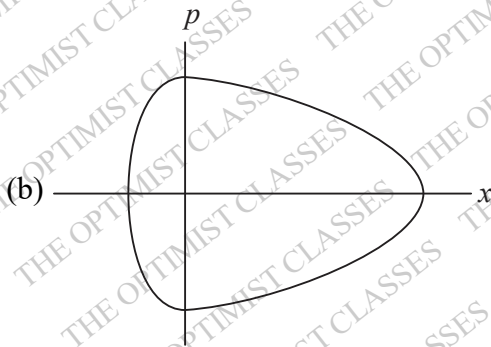
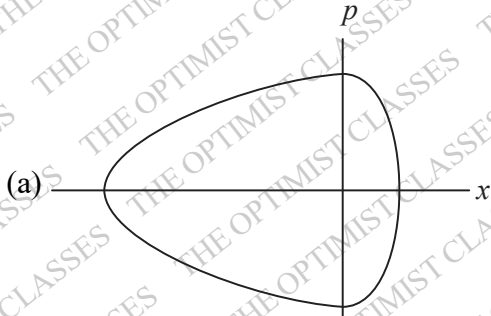
$$(d) \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

60. An amplifier with a voltage gain of 40dB without feedback is used in an electronic circuit. A negative feedback with a fraction $1/40$ is connected to the input of this amplifier, the net gain of the amplifier in the circuit is closest to
- (a) 40dB (b) 37dB
(c) 29dB (d) 20dB
61. The Lagrangian of a system of two particles is

$$L = \frac{1}{2} \dot{x}_1^2 + 2\dot{x}_2^2 - \frac{1}{2} (x_1^2 + x_2^2 + x_1 x_2). \text{ The normal frequencies are best approximated by}$$

- (a) 1.2 and 0.7 (b) 1.5 and 0.5
(c) 1.7 and 0.5 (d) 1.0 and 0.4
62. The Lagrangian of a particle in one dimension

$L = \frac{m}{2} \dot{x}^2 - ax^2 - V_0 e^{-10x}$ where a and V_0 are positive constants. The best qualitative representation of a trajectory in the phase space is



thermal equilibrium at temperature T in a constant magnetic field B . The average magnetic moment

$\langle M \rangle$, as a function of $\frac{k_B T}{\mu_B B}$, is best represented

by

(a) $\langle M \rangle$

(b) $\langle M \rangle$

(c) $\langle M \rangle$

(d) $\langle M \rangle$

63. A paramagnetic salt with magnetic moment per ion $\mu_{\pm} = \pm \mu_B$ (where μ_B is the Bohr magneton) is in

64. Two small metallic objects are embedded in a weakly conducting medium of conductivity σ and dielectric constant ϵ . A battery connected between them leads to a potential difference V_0 . It is subsequently disconnected at time $t=0$. The potential difference at a later time t is

(a) $V_0 e^{-\frac{t\sigma}{4\epsilon}}$ (b) $V_0 e^{-\frac{t\sigma}{2\epsilon}}$
 (c) $V_0 e^{-\frac{2t\sigma}{4\epsilon}}$ (d) $V_0 e^{-\frac{t\sigma}{\epsilon}}$

65. Earth may be assumed to be an axially symmetric freely rotating rigid body. The ratio of the principal moments of inertia about the axis of symmetry and an axis perpendicular to it is 33:32. If T_0 is the time taken by earth to make one rotation around its axis of symmetry, then the time period of precession is closest to

(a) $33T_0$ (b) $33T_0 / 2$
 (c) $32T_0$ (d) $16T_0$

66. In the absorption spectrum of H-atom, the frequency of transition from the ground state to the first excited state is ν_H . The corresponding frequency for a bound state of a positive charged muon (μ^+) and an electron is ν_μ . Using

$$m_\mu = 10^{-28} \text{ kg}, m_e = 10^{-30} \text{ kg} \text{ and } m_p \gg m_e, m_\mu,$$

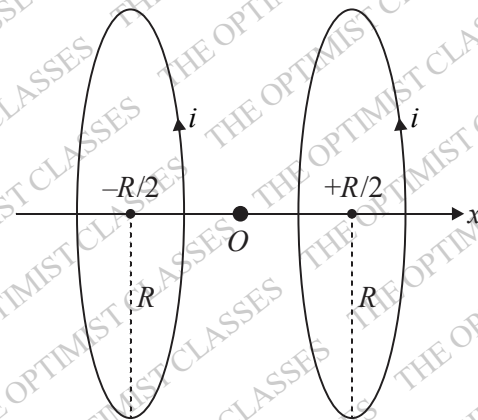
the value of $(\nu_\mu - \nu_H) / \nu_H$ is

(a) 0.001 (b) -0.001
 (c) -0.01 (d) 0.01

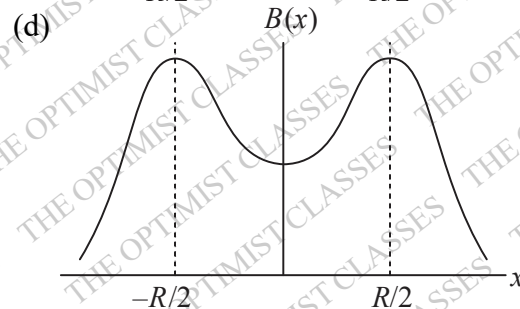
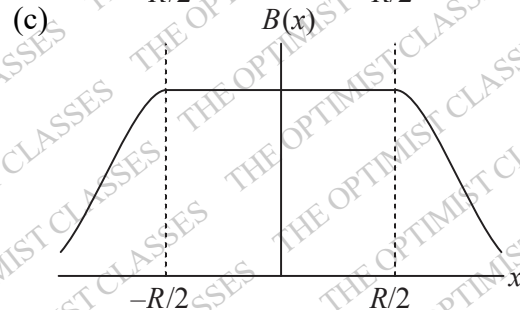
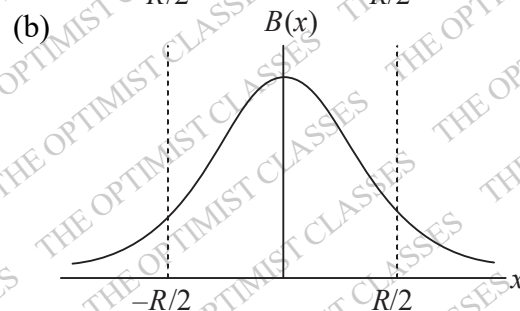
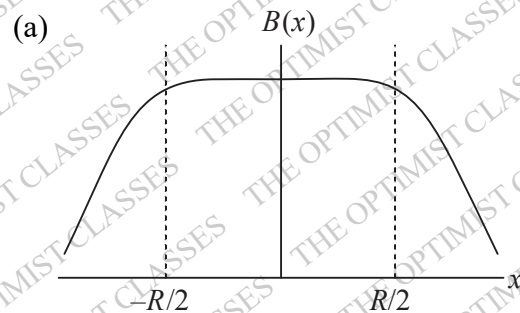
67. A bucket contains 6 red and 4 blue balls. A ball is taken out of the bucket at random and two balls of the same colour are put back. This step is repeated once more. The probability that the numbers of red and blue balls are equal at the end, is

(a) $\frac{4}{11}$ (b) $\frac{2}{11}$
 (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

68. Two parallel conducting rings, both of radius R , are separated by a distance R . The planes of the rings are perpendicular to the line joining their centres, which is taken to be the x -axis.



If both the rings carry the same current i along the same direction, the magnitude of the magnetic field along the x -axis is best represented by



69. The value of the integral $\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2 + 1} dx$ for $\alpha > 0$

is

- (a) πe^{α} (b) $\pi e^{-\alpha}$
 (c) $\pi e^{-\alpha/2}$ (d) $\pi e^{\alpha/2}$

70. The Raman rotational-vibrational spectrum of introgen molecules is observed using an incident radiation of wavenumber 12500 cm^{-1} . In the first shifted band, the wavenumbers of the observed lines (in cm^{-1}) are 10150, 10158, 10170, 10182 and 10190. The values of vibrational frequency and rotational constant (in cm^{-1}), respectively, are

- (a) 2330 and 2 (b) 2350 and 2
 (c) 2350 and 3 (d) 2330 and 3

71. A system of N non-interacting particles in one-dimension, each of which is in a potential $V(x) = gx^6$ where $g > 0$ is a constant and x denotes the displacement of the particle from its equilibrium position. In thermal equilibrium, the heat capacity at constant volume is

- (a) $\frac{7}{6} Nk_B$ (b) $\frac{4}{3} Nk_B$
 (c) $\frac{3}{2} Nk_B$ (d) $\frac{2}{3} Nk_B$

72. The elastic scattering process $\pi^- p \rightarrow \pi^- p$ may be treated as a hard-sphere scattering. The mass of π^- , $m_{\pi} \approx \frac{1}{6} m_p$, where $m_p \approx 983 \text{ MeV} / c^2$ is the mass of the proton. The total scattering cross-section is closest to

- (a) 0.01 milli-barn (b) 1 milli-barn
 (c) 0.1 barn (d) 10 barn

73. The Hamiltonian for a spin-1/2 particle in a magnetic field $B = B_0 \hat{k}$ is given by $H = \lambda S \cdot B$, where S is its spin (in units of \hbar) and λ is a constant. If the average spin density is $\langle S \rangle$ for an ensemble of such non-interacting particles, then

$$\frac{d}{dt} \langle S_x \rangle$$

(a) $\frac{\lambda}{\hbar} B_0 \langle S_x \rangle$

(b) $\frac{\lambda}{\hbar} B_0 \langle S_y \rangle$

(c) $-\frac{\lambda}{\hbar} B_0 \langle S_x \rangle$

(d) $-\frac{\lambda}{\hbar} B_0 \langle S_y \rangle$

74. To first order in perturbation theory, the energy of the ground state of the Hamiltonian

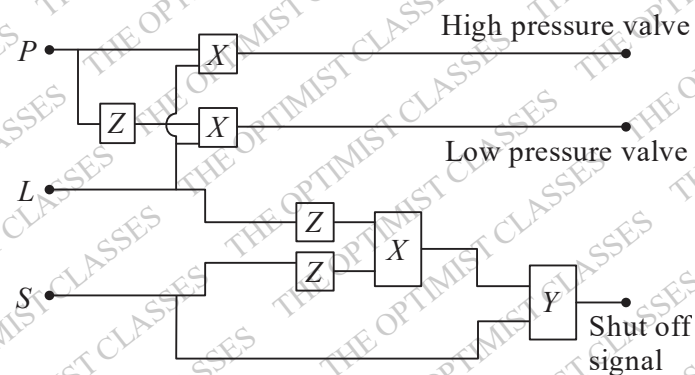
$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 + \frac{\hbar \omega}{\sqrt{512}} \exp\left[-\frac{m \omega}{\hbar} x^2\right]$$

(treating the third term of the Hamiltonian as a perturbation) is

(a) $\frac{15}{32} \hbar \omega$ (b) $\frac{17}{32} \hbar \omega$

(c) $\frac{19}{32} \hbar \omega$ (d) $\frac{21}{32} \hbar \omega$

75. A liquid oxygen cylinder system is fitted with a level-sensor (L) and a pressure-sensor (P), as shown in the figure below. The outputs of L and P are set to logic high ($S = 1$) when the measured values exceed the respective preset threshold values. The system can be shut off either by an operator by setting the input S to high, or when the level of oxygen in the tank falls below the threshold value.



The logic gates X, Y and Z, respectively, are

- (a) OR, AND and NOT
 (b) AND, OR and NOT
 (c) NAND, OR and NOT
 (d) NOR, AND and NOT

ANSWER KEY

- | | | |
|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (a) |
| 4. (a) | 5. (b) | 6. (a) |
| 7. (d) | 8. (c) | 9. (a) |
| 10. (b) | 11. (a) | 12. (a) |
| 13. (d) | 14. (b) | 15. (a) |
| 16. (a) | 17. (d) | 18. (a) |
| 19. (a) | 20. (b) | 21. (d) |
| 22. (d) | 23. (c) | 24. (b) |
| 25. (a) | 26. (b) | 27. (a) |
| 28. (d) | 29. (a) | 30. (b) |
| 31. (b) | 32. (b) | 33. (b) |
| 34. (a) | 35. (b) | 36. (a) |
| 37. (b) | 38. (c) | 39. (a) |
| 40. (d) | 41. (d) | 42. (b) |
| 43. (d) | 44. (c) | 45. (c) |
| 46. (*) | 47. (d) | 48. (b) |
| 49. (c) | 50. (d) | 51. (a) |
| 52. (b) | 53. (a) | 54. (d) |
| 55. (b) | 56. (d) | 57. (c) |
| 58. (b) | 59. (a) | 60. (c) |
| 61. (d) | 62. (b) | 63. (c) |
| 64. (d) | 65. (c) | 66. (c) |
| 67. (b) | 68. (a) | 69. (b) |
| 70. (a) | 71. (d) | 72. (c) |
| 73. (d) | 74. (b) | 75. (b) |