THE OPTIMIST CLASSES
IIT-JAM TOPPERS

8 AIR
MANOJ KUMAR SINGH

9 AIR
ABHAY

12 AIR
PAWAN

15 AIR
SATYAM

21 AIR
SOUMIL GIRISH SAHU

29 AIR
BHOOMIJA

31 AIR
AKSHIT AGGARWAL

32 AIR
SHIKHAR CHAMOLI

33 AIR
RAVI SINGH ADHIKARI

44 AIR
GAURAV JHA

56 AIR
SWAPNIL JOSHI

65 AIR
LOKESH BHATT

75 AIR
GOPESH VISHVAKARMA

77 AIR
VAIBHAV

80 AIR
SHASWAT CHAMOLI

CSIR-NET-JRF RESULTS 2022

102 AIR — JRF
ANNU
DL01000308

133 AIR — JRF
ALANKAR
UP15000162

155 AIR — JRF
SAHIL RANA
HR09000108

168 AIR — JRF
JAYESTHI
RJ11000161

174 AIR — JRF
DASRATH
RJ06000682

181 AIR — JRF
VIVEK
UK01000439

183 AIR — JRF
UZAIR AHMED
UP02000246

191 AIR — JRF
SURYA PRATAP SINGH
RJ06000232

195 AIR — JRF
HIMANSHU
UP10000095

222 AIR — JRF
CHANDAN
RJ09000159

228 AIR — JRF
SAIKHOM JOHNSON
MN01000196

231 AIR — JRF
AJAY SAINI
RJ06001744

258 AIR — JRF
VIKAS YADAV
RJ06001102

283 AIR — JRF
JYOTSNA KOHLI
UK02000262

318 AIR — JRF
SHYAM SUNDAR
RJ06000615

THE OPTIMIST CLASSES

AN INSTITUTE FOR NET-JRF/GATE/IIT-JAM/JEST/TIFR/M.Sc ENTRANCE EXAMS

CONTACT: 9871044043

TATA INSTITUTE OF FUNDAMENTAL RESEARCH (TIFR)
PREVIOUS YEAR QUESTIONS
YEAR 2012

Name :- .... Ref. Code :- .... Date :- ....

Please read all instructions carefully before you attempt the questions

1. Please fill-in details about name,reference code etc. on the question paper and answer sheet. Answer sheet is machine-readable. Read the instructions given on the reverse of the answer sheet before you start filling it up. Use only HB pencils to fill-in the answer sheet.

2. Indicate your ANSWER ON THE ANSWER SHEET by blackening the appropriate circle for each questions. *Do not mark more than one circle for any question*: this will be treated as a wrong answer.

3. This test comes in two sections, **Section A** and **Section B** , both of which contain multiple choice-type questions. Only ONE of the options given at the end of each question is correct. Section A contains 20 questions , each with 4 options, and Section B contains 8 questions, each with 6 options. The maximum marks are 60 for section A plus 40 for Section B, totaling to 100. Marking shall be as:

   (i) If the answer is **correct** : +3 marks in Section A: +5 marks in Section B.

   (ii) If the answer is **incorrect** : -1 mark in both Section A & B.

   (iii) If the answer is not **attempted**: 0 marks in both Section A & B.

   (iv) If more than one box is **marked**: 0 marks in both Section A & B.

   Note that negative marking as indicated above will be implemented.

4. As a rough guideline, the time spent on questions in Section A should be about 5 minutes each; questions in Section B should take about 10 minutes each. Obviously, some questions may take a little less time while others may require a litle more.

5. We advise you to first mark the correct answer on the Question PAPER and then to TRANSFER these to the ANSWER SHEET only when you are sure of your choice.

6. Rough work may be done on blank pages of the question paper. If needed, you may ask for extra rough sheets from an invigilator.

7. **Use of calculators is permitted**. Calculator which plots graphs is NOT allowed. Multiple-use devices such as cell phones, smart phones etc., CANNOT be used for this purpose.

8. Do NOT ask for clarifications from the invigilators regarding the questions. They have been instructed not to respond any such inquiries from candidates. In case a Correction/clarification is deemed necessary; the invigilator(s) will announce it publicly.

SECTION :- A

$20 \times 3 = 60$ MARKS

A1.　Two different 2×2 matrices A and B are found to have the same eigenvalues. It is then correct to states that $A = SBS^{-1}$ where S can be a

(a) traceless 2×2 matrix　　　　　　　　　　(b) Hermitian 2×2 matrix

(c) unitary 2×2 matrix　　　　　　　　　　(d) arbitrary 2×2 matrix

A2.　The function $f(x)$ represents the nearest integer less than x, e.g.

$$f(3.14) = 3.$$

The derivative of this function (for arbitrary x) will be given in terms of the integers n as $f'(x) =$

(a) 0　　　　　(b) $\Sigma_n \delta(x-n)$　　　　　(c) $\Sigma_n |x-n|$　　　　　(d) $\Sigma_n f(x-n)$

A3.　Two masses M_1 and M_2 $(M_1 < M_2)$ are suspended from a perfectly rigid horizontal support by a system of three taut massless wires W_1, W_2 and W_3, as shown in the figure. All the three wires have identical cross-sections and elestic properties and are known to be very strong.
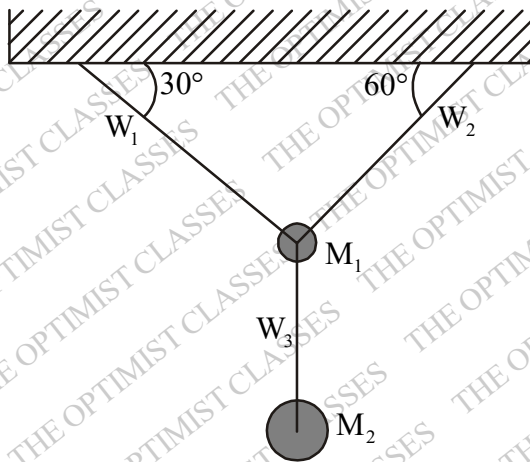
If the mass M_2 is increased gradually, but without limit, we should expect the wires to break in the following order:

(a) first W_2, then W_1　　　　　　　　　　(b) first W_1, then W_2

(c) first W_2, then W_3　　　　　　　　　　(c) first W_3

A4.　A high-velocity missile, travelling in a horizontal line with a kinetic energy of 3.0 Giga-Joules (GJ), explodes in flight and breaks into two pieces A and B of equal mass . One of these pieces (A) flies off in a straight line perpendicular to the original direction in which the missile was moving and its kinetic energy is found to be 2.0 GJ. If gravity can be neglected for such high-velocity projectiles, it follows that the other piece (B) flew off in a direction at an angle with the original direction of

(a) 30° (b) 33°24′ (c) 45° (d) 60°

A5. Consider a spherical planet, rotating about an axis passing through its centre. The velocity of a point on its equator is v_{eq}. If the acceleration due to gravity g measured at the equator is half of the value of g measured at one of the poles, then the escape velocity for a particle shot upwards from that pole will be

(a) $v_{eq}/2$ (b) $v_{eq}/\sqrt{2}$ (c) $\sqrt{2}\, v_{eq}$ (d) $2\, v_{eq}$

A6. A dynamical system with two degrees of freedom, has generalized coordinate q_1 and q_2, and kinetic energy

$$T = \lambda \dot{q}_1 \dot{q}_2$$

If the potential energy is $V(q_1, q_2) = 0$, the correct form of the Hamiltonian for this system is

(a) $p_1 p_2 / \lambda$

(b) $\lambda \dot{q}_1 \dot{q}_2$

(c) $\left(p_1 \dot{q}_1 + p_2 \dot{q}_2 \right)/2$

(d) $\left(p_1 q_2 + p_2 q_1 \right)/2$

A7. An ideal liquid of density 1 gm/cc is flowing at a rate of 10 gm/s through a tube with varying cross-section, as shown in the figure.
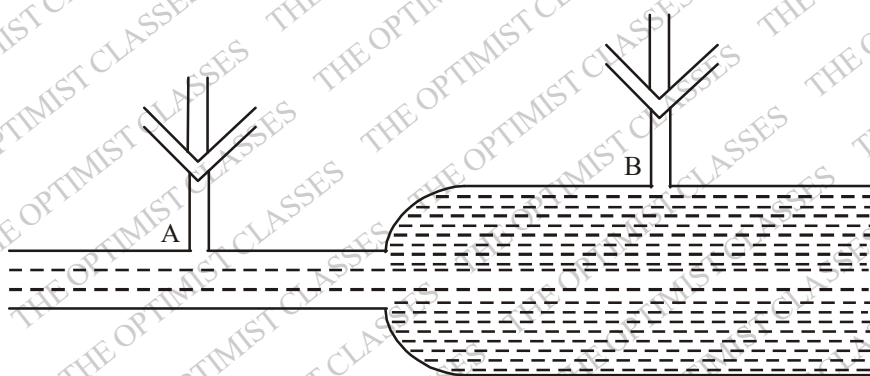
Two pressure gauges attached at the points A and B (see figure) show readings of P_A and P_B respectively. If the radius of the tube at the points A and B is 0.2 cm and 1.0 cm respectively, then the difference in pressure $(P_B - P_A)$, in units of dyne cm^{-2}, is closest to

(a) 100 (b) 120 (c) 140 (d) 160

A8. Unpolarised light of intensity I_0 passes successively through two identical linear polarisers A and B, placed such that their polarisation axes are at an angle of $45°$ (see figure) with respect to one another.
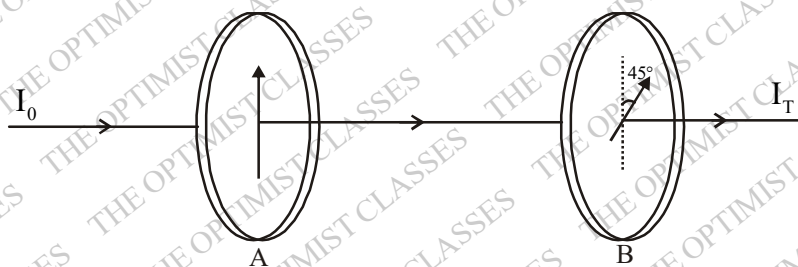
Assuming A and B to be perfect polarisers (i.e., no absorption losses), the intensity of the transmitted light will be $I_T =$

(a) $I_0 / 4$ (b) $I_0 / 2\sqrt{2}$ (c) $I_0 / 2$ (d) $I_0 / \sqrt{2}$

A9. Three equal charges Q are successively brought from infinity and each is placed at one of the three vertices of an equilateral triangles. Assuming the rest of the Universe as a whole to be neutral, the energy E_0 of the electrostatics field will increase, successively, to

$$E_0 + \Delta_1, \qquad\qquad E_0 + \Delta_1 + \Delta_2 \qquad\qquad E_0 + \Delta_1 + \Delta_2 + \Delta_3$$

Where $\Delta_1 : \Delta_2 : \Delta_3 =$

(a) $1 : 2 : 3$ (b) $1 : 1 : 1$ (c) $0 : 1 : 1$ (d) $0 : 1 : 2$

A10. Five sides of a hallow metallic cube are grounded and the sixth side is insulated from the rest and is held at a potential Φ (see figure).
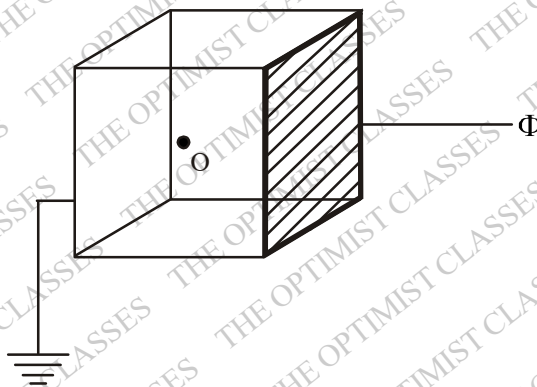
The potential at the center of the cube is

(a) 0 (b) $\Phi / 6$ (c) $\Phi / 5$ (d) $2\Phi / 3$

A11. Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers conatins an ideal gas, while the other half is a vaccum (see figure).
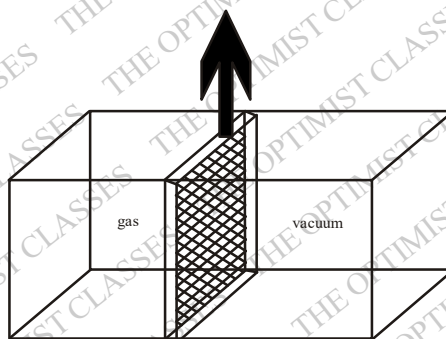
If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy

per molecule of the system will increase by an amount

(a) $2k_B$ (b) $k_B \ell n(1/2)$ (c) $k_B \ell n(2)$ (d) $k_B \ell n(2)/2$

A12. When a gas is enclosed in an impermeable box and heated to a high temperature T, some of the neutral atoms lose an electron and become ions. If the number density of neutral atoms, ions and electrons is N_a, N_i and N_e respectively, these can be related to the average volume V_a occupied by an atoms/ion and the ionisation energy E by the relation

(a) $N_e(N_a + N_i) = (N_a/V_a)\exp(-E/k_B T)$ (b) $N_a(N_e + N_i) = (N_a/V_a)\exp(-E/k_B T)$

(c) $N_e N_i = (N_a/V_a)\exp(+E/k_B T)$ (d) $N_e N_i = (N_a/V_a)\exp(-E/k_B T)$

A13. In a scanning tunnelling microscope, a fine Platinum needle is held close to a metallic surface in vacuum and electrons are allowed to tunnel across the tiny gap δ between the surface and the needle. The tunnelling current I is related to the gap δ, through positive constants a and b, as

(a) $I = a - b\delta$ (b) $I = a + b\delta$

(c) $\log I = a - b\delta$ (d) $\log I = a + b\delta$

A14. A particle in a one-dimensional potential has the wavefunction

$$\varphi(x) = \frac{1}{\sqrt{a}}\exp\left(\frac{-|x|}{a}\right)$$

where a is constant. It follows that for a positive constant V_0, the potential $V(x) =$

(a) $V_0 x^2$ (b) $V_0|x|$ (c) $-V_0\delta(x)$ (d) $-V_0/|x|$

A15. Consider the high excited states of a hydrogen atom corresponding to large value of the principal quautum number $(n \gg 1)$. The wavelength λ of a photon emitted due to an electron undergoing a transition between two such states with consecutive values of n (i.e. $\psi_{n+1} \rightarrow \psi_n$) is related to the wavelength λ_α of the K_α line of hydrogen by

(a) $\lambda = \frac{n^3}{8}\lambda_\alpha$ (b) $\lambda = \frac{3n^3}{8}\lambda_\alpha$ (c) $\lambda = n^2\lambda_\alpha$ (d) $\lambda = \frac{4}{n^2}\lambda_\alpha$

A16. A proton is accelerated to a high energy E and shot at a nucleus of Oxygen $\binom{16}{8}O$. In order to penetrate the Coulomb barrier and reach the surface of the Oxygen nucleus, E must be at least
(a) 3.6 MeV (b) 1.8 MeV (c) 45 keV (d) 180 eV

A17. A monochromatic beam of X-rays with wavelength λ is incident at an angle θ on a crystal with lattice spacings a and b as sketched in the figure below.
A condition for there to be a maximum in the diffracted X-ray intensity is

(a) $2\sqrt{a^2 + b^2}\,\sin\theta = \lambda$

(b) $2b\cos\theta = \lambda$

(c) $2a\cos\theta = \lambda$

(d) $(a+b)\sin\theta = \lambda$
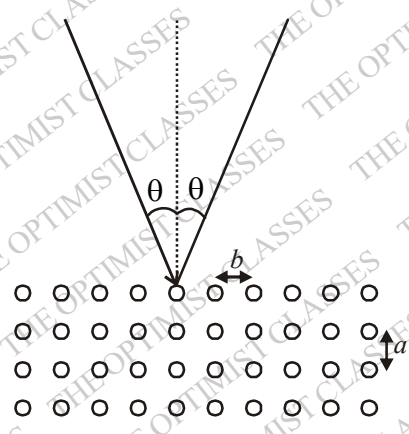
A18. Suppose the energy band diagram of a certain pure crystalline solid is as shown in the figure below, where the energy (E) varies with crystal momentum (k) as $E \propto k^2$.
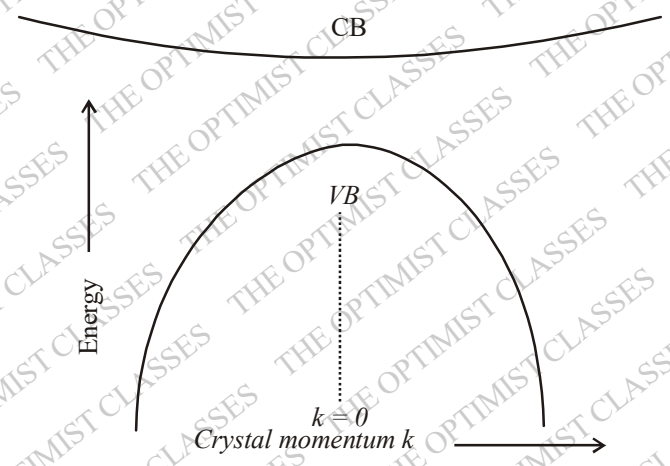

At finite temperatures the bottom of the conduction band (CB) is partially filled with electrons (e) and the top of the valence band (VB) is partially filled with holes (h). If an electric filed is applied to this solids, both e and h will start moving. If the time between collisions is the same for both e and h, then

(a) e and h will move with the same speed in opposite directions

(b) h will on an average achieve higher speed than e

(c) e will on an average achieve higher speed than h

(d) e and h will recombine and after a while there will no flow of charges
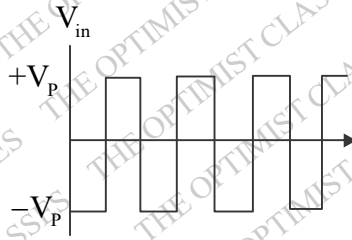
A19. Consider the circuit shown below.
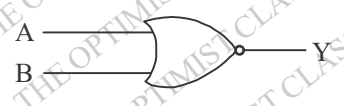
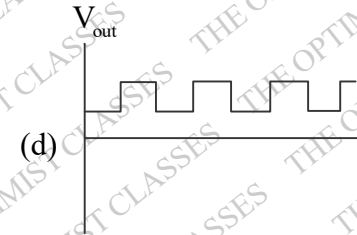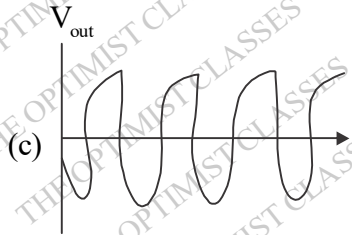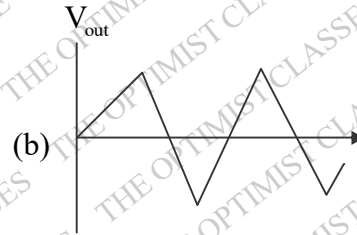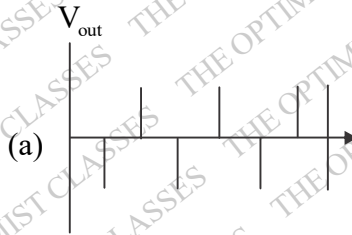The minimum number of NAND gates required to design this circuit is

(a) 6   (b) 5   (c) 4   (d) 3

A20. Consider the following circuit :
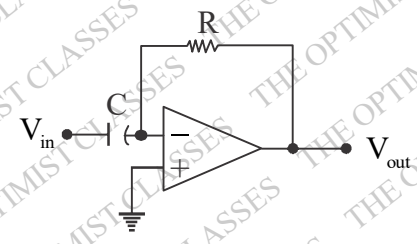
If the waveform given below is fed in at V_{in}

then the waveform at the output V_{out} will be

(a)

(b)

(c)

(d)

SECTION :- B

$10 \times 4 = 40$ MARKS

B1. Consider the integral

$$\int_{-p^2}^{+p^2} \frac{dx}{\sqrt{x^2 - p^2}}$$

where p is a constant. This integral has a real, non singular value of

(a) $p < -1$ (b) $p > 1$ (c) $p = 1$ (d) $p \to 0$ (e) $p \to \infty$

B2. If we model the electron as a uniform sphere of radius r_e, spinning uniformly about an axis passing through its centre with angular momentum $L_e = \hbar/2$, and demand that the velocity of rotation at the equator cannot exced the velocity c of light in vacuum, then the minimum value of r_e is

(a) 19.2 fm (b) 0.192 fm (c) 4.8 fm (d) 1960 fm (e) 480 fm

B3. The intensity of light coming from a distant star is measured using two identical instruments A and B, where A is placed in a satellite outside the Earth's atmosphere, and B is placed on the Earth's surface. The results are as follows :

Colour	Wavelength (nm)	Intensity at A (nano Watts)	Intensity at B (nano Watts)
green	500	100	50
red	700	200	x

Assuming that there is scattering, but no absorption of light in the Earth's atmosphere at these wavelengths, the value of x can be estimated as

(a) 137 (b) 147 (c) 157 (d) 167 (e) 177

B4. Consider three identical infinite straight wires A, B and C arranged in parallel on a plane as shown in the figure.
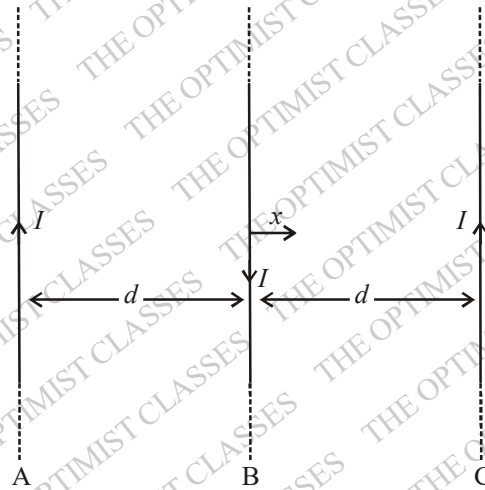
The wires carry equal currents I with directions as shown in the figure and have mass per unit lenght m. If the wires A and C are held fixed and the wire B is displaced by a small distance x from its position, then it (B) will execute simple harmonic motion with a time period

(a) $2\pi \sqrt{\dfrac{m}{\pi \mu_0}} \left(\dfrac{d}{I} \right)$

(b) $2\pi \sqrt{\dfrac{2\pi m}{\mu_0}} \left(\dfrac{d}{I} \right)$

(c) $2\pi \sqrt{\dfrac{\pi m}{\mu_0}} \left(\dfrac{d}{I} \right)$

(d) $2\pi \sqrt{\dfrac{m}{2\pi \mu_0}} \left(\dfrac{d}{I} \right)$

(e) $2\pi \sqrt{\dfrac{m}{\mu_0}} \left(\dfrac{d}{I} \right)$

B5. The normalized wavefunction of a Hydrogen atom are denoted by $\psi_{n,\ell,m}(\vec{x})$, where n, ℓ and m are, respecstively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\psi(\vec{x}) = \frac{1}{3}\psi_{1,0,0}(\vec{x}) + \frac{2}{3}\psi_{2,1,0}(\vec{x}) + \frac{2}{3}\psi_{3,2,-2}(\vec{x})$$

The expectation value $\langle E \rangle$ of the energy of this electron, in electron -Volts (eV) will be approximately

(a)   -1.5   (b) -3.7   (c) -13.6   (d) -80.1   (e) +13.6

B6.   The strongest three lines in the emission spectrum of an intersteller gas cloud are found to have wavelengths $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like

(a) free particles   (b) particles in a box   (c) harmonic oscillators
(d) rigid rotators   (e) hydrogenic atoms

B7.   When light is emitted from a gas of excited atoms, the lines in the spectrum are Doppler-broadened due to the thermal motion of the emitting atoms.

The Doppler width of an emission line of wavelength 500 nanometres (nm) emitted by an excited atom of Argon $\left(_{20}^{40}A\right)$ at room temperature $\left(27^\circ C\right)$ can be estimated as

(a) 5.8×10^{-4} nm   (b) 3.2×10^{-4} nm   (c) 3.2×10^{-3} nm

(d) 2.5×10^{-3} nm   (e) 1.4×10^{-3} nm

B8.   In a nuclear reactor, Plutonium $\left(_{94}^{239}pu\right)$ is used as fuel, releasing energy by its fission into isotopes of Barium $\left(_{54}^{146}Ba\right)$ and Strontium $\left(_{38}^{91}Sr\right)$ through the reaction

$$_{94}^{239}Pu + _{0}^{1}n \rightarrow _{56}^{146}Ba + _{38}^{91}Sr + 3 \times _{0}^{1}n$$

The binding energy ($B.E.$) per nucleon of each of these nuclides is given in the table below:

| Nuclide	$_{94}^{239}$Pu	$_{54}^{146}$Ba	$_{38}^{91}$Sr
B.E. per nucleon (MeV)	7.6	8.2	8.6

Using this information, one can estimate the number of such fission reactions per second in a 100 MW reactor as

(a) 3.9×10^{18} (b) 7.8×10^{18} (c) 5.2×10^{19} (d) 5.2×10^{18} (e) 8.9×10^{17}

B9.   Metallic Copper is known to form cubic crystal and the lattice constant is measured from X-ray diffraction studies to be about 0.36 nm. If the specific gravity of Copper is 8.96 and its atomic weight is 63.5, one can conclude that
(a) the crystals are of simple cubic type
(b) the crystals are of b.c.c. type
(c) the crystals are of f.c.c. type
(d) the crystals are a mixture of f.c.c. and b.c.c. types
(e) there is insufficient data to distinguish between the previous options

B10.   The voltage regulator circuit shown in the figure has been made with a Zener diode rated at 15V, 200mW. It is required that the circuit should dissipate 150mW power across the fixed load resistor R_L.
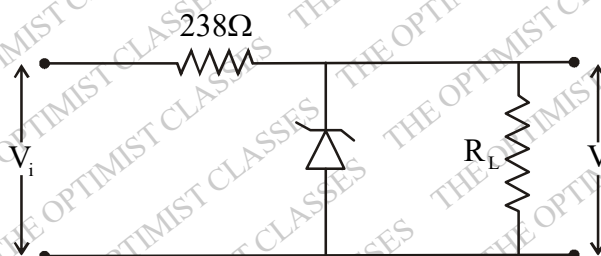
For stable operation of this circuit, the input voltage V_i must have a range

(a) 17.5 V - 20.5 V (b) 15.5 V - 20.5 V (c) 15.5 V - 22.5 V

(d) 17.5 V - 22.5 V (e) 15.0 V - 22.5 V

ANSWER KEY

SECTION-A

1.	(c)	2.	(b)	3.	(c)	4.	(a)	5.	(d)	6.	(a)
7.	(b)	8.	(a)	9.	(d)	10.	(b)	11.	(c)	12.	(d)
13.	(c)	14.	(c)	15.	(b)	16.	(a)	17.	(b)	18.	(b)
19.	(c)	20.	(a)										

SECTION-B

B1.	(d)	B2.	(e)	B3.	(d)	B4.	(c)	B5.	(b)	B6.	(d)
B7.	(e)	B8.	(a)	B9.	(c)	B10.	(a)				